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Francis Douce.

THE

Grounde of Artes:

teaching the work and pra-
ctise of Arithmetike, bothe in whole numbers and Fractions, after a more easyer and exacter sorte than any like hath hither-to bin set forth:

Made by M. ROBERT RECORD, *Doctor in Physike, and now of late diligently* ouerseene and augmented with newe and necessarie Additions.

I. D.

That vvhiche my freende hath vvell begonne,
For very loue to common vveale,
Neede not all vvhole to be nevve donne,
But nevve encreasse I doe reueale.

Some thyng herein, I once redresse,
And novve agayne for thy behoofe,
Of zeale I doe, and at requeste,
Bothe mende and adde, fitte for all proofe.

Of Numbers vse, the endlesse might,
No vvitte nor language can expresse,
Applye and Trie, bothe day and night,
And then this truthe thou vvilt confesse.

LONDINI,
Anno Domini. 1575.

Sola salus seruire Deo

The bookes Verdict.

To please or displease sure I am,
But not of one sorte to euery man:
To please the beste sorte would I fayne,
The frowarde displease shall I certayne.
Yet wishe I will, though not with hope,
All eares and mouthes to please or stoppe.

TO THE MOSTE mightie Prince Edwarde the sixte, by the grace of God, *King of Englande, Fraunce, and Irelande. &c.*

The excellencie of mannes nature, beeing suche, as it is by Gods diuine fauour (most mightie Prince) not onely created in highnesse of degree farre aboue all other corporall thynges, but by perfection of reason and searche of witte much approching towarde the image of God, as not onely the holy Scriptures doe testifie, but also those naturall Philosophers, which exactely didde consider the nature of man, and namely the farre reache and infinite compasse of the workes of the mynde, were inforced to confesse, that man scarcely was able to knowe himselfe. And if he would duly ponder the nature of himselfe, he shoulde fynde it so strange, that it myght seeme vnto him a verie miracle.

And therof sprang that saying: magnum miraculum est homo, maximum miraculum sapiens homo. For vndoubtedly as man is one of the greatest miracles that euer God wrought, so a wyse man is playnly the greatest.

And therfore was it that some did account the head of a man to be the greatest miracle in the worlde, bicause not onely of the straunge workemanship that is in it, but muche more of the efficacie of reason, wit, memorie, imagination, and suche other powers and workes of the mynd, whiche can more easily conceyue any thing in a maner, than vnderstand it selfe. And amongst all the creatures of God it fyndeth none more difficulte to be perceyued than these same powers of it selfe, whereby it dothe conceyue and iudge, as it may be well coniectured by the diuersytie of opinyons that the wysest Philosophers did vtter, touchyng the spirit of man and the substance of it: whereof at this present I intende to make no rehersall, but who so listeth to reade therof, maye fynde it largely set forth not only in Aristotle his bookes De anima, but also in Galene hys booke called Historia philosophica, and again in Plutarche his worke, De philosophorum placitis, whose wordes are also repeated of Eusebius

sebius in the xv. booke, τῆς εὐαγγελικῆς προπαρασκευῆς, vnto whome I remit them that haue desire to vnderstande the intricate difficultie of knowing oure owne selues, as touching our beste parte, and that parte whereby we deserue to beare the name of men.

This matter seemed so obscure and difficulte, in knowledge, that Galen, who for his excellent wisdome and iudgement in naturall works, is called of manye men a Miracle in nature, yet in searching the nature and substance of the spirite of man, hee not onely confesseth himselfe ignorant, but counteth plaine temerity to attempte too finde it: so far aboue ẏ hope of mans knowledge is that parte wherby mā doth know and iudge of things. And although the ignorant sorte (whiche hate all things that they know not (do litle esteeme the profoundnesse of mannes spirit, and of reason, the chiefe power and facultie of it, yet as there is a kynde of feare and obedience of all vnreasonable beastes vnto man by the working power of God, so is there in those small reasoned persons a certayne kynde of reuerence towarde wysedome and reason, whiche they do shewe oftentymes, and by power of persuasion, are inforced to obeye reason, will they

wyll they.

And hereby came it to passe, that the rudenesse of the fyrst age of man was brought vnto some more ciuil trade, as it is well declared by Cicero in the beginning of his fyrst booke *De inuentione Rhetorica*, where he sayth thus:

Nam fuit quoddam tempus quum in agris homines paſsim beſtiarum more vagabantur, & ſibi victu ferino vitam propagabāt, nec ratione animi quicquam, ſed plæraq; viribus corporis adminiſtrabant. Non dum diuinę religionis, non humani officij ratio colebatur. Nemo legitimas viderat nuptias, non certos quiſquam inſpexerat liberos, non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atque inſcitiam cæca ac temeraria dominatrix animi cupiditas, ad ſe explendam viribus corporis abutebatur, pernicioſiſsimis ſatellitibus. Quo tempore quidam, magnus videlicet vir & ſapiens, cognouit quæ materia eſſet, & quanta ad maximas res oportunitas in animis ineſſet hominum

num

num, si quis eam posset elicere, & præcipiendo meliorem reddere. Qui dispersos homines in agris, & in tectis syluestribus abditos, ratione quadam compulit in vnum locum, & congregauit: & eos in vnamquanque rem inducens vtilem atq; honestam, primo propter insolentiã reclamantes, deinde propter rationem atque orationem studiosius audientes, ex feris & immanibus mites reddidit, & mansuetos.

This long repetition of Tullies wordes will seme tedious to thẽ which loue but little, and care much lesse for the knowledge of reason, but vnto your Maiestie (I dare say) it is a delectable remẽbrance, and vnto me it semed so pleasant, that I coulde scarse staye my penne from writing all that myne eies did so greedily reade.

This sentence of Cicero am I lothe to translate into Englishe, partly for that vnto your Maiestie it needeth no translation, but especially knowing how far ẏ grace of Tullies eloquence doth excell any English mans tongue, and muche more exceedeth the basenesse of my barbarous style, yet for the fruite of the sentence, I had rather vnto my meere

Englische countrie men vtter the rudenesse of my translation, than to defraude them the benefite of so good a lesson, trusting, they will so learne to loue reason, that they wil also gladly and greedily imbrace all good sciences that maye helpe to the iuste furniture of the same, when they consyder that infourmed reason was the onelye instrumente, or at leaste the chiefe meane to bryng men vnto Ciuile regiment, from barbarous manners and beastlye conditions.

For the tyme was (sayth Tullie) that men wandred abroade in the fieldes vp and downe like beastes, and vsed no better order in feeding than they, so that by reasons rule they wrought nothing, but most of their doyng did they atchieue by force of strengthe. At this time there was no iuste regard of religion toward God, nor of duetie toward man. No man had seene righte vse of mariage, neyther did any manne knowe their owne children from other, nor no man had felte the commoditie of iust Lawes: so that through errour and ignoraunce, wylfull lust, lyke a blynde and headie ruler, abused bodilye strengthe as a moste mortall minister for the satisfying of his desyre. At that tyme was there one, whiche not only in power, but

also in wisedome was greate, And he conside-red howe that in the myndes of men was bothe apt instrumentes, and greate occasion to the due accomplishement of moste weyghtie affaires, if a man coulde applye them to vse, and by teaching of rules, frame them to better trade. This man with perswasion of reason gathered into one place the people that were wandering about the fieldes, and laye lurking in wylde cotages and woddes: And bryngyng thē into one common societye, dyd trade them to all suche thinges, as eyther were profitable or honest, although not without repining at the first, by reason that they had not beene so accustomed before. Yet at lengthe thorowghe reason and persuasyon of wordes they obeyed hym more dilygently, and so of a wylde and cruell people, hee made them curteous and gentle.

Thus hath Tullie sette forthe the efficacie of reason and persuasion, howe it was able to conuerte wylde people to a myldenesse, and to chaunge their furious cruelnesse into gentle curtesye: were it not nowe a great reproch in thys oure tyme (when knowledge raigneth so large) that menne shoulde shewe themselues lesse obsequious to reason? Unlesse it maye

bee thought , that nowe euerye manne ha-
uing sufficient knowledge of him selfe, nee-
deth not to hearken to the perswasion of o-
thers.

In dede he that thinketh himselfe wise, will not esteme the reason of any other, be he neuer so wise, so that of such one it may wel be said: Hee that thinketh himselfe wyser than hee is, may iustly be counted a double foole: wherfore suche men are not to be permitted in open audience to talke, but must be put to silence, and made to giue eare to reason, which reason consisteth not in a multitude of wordes, heaped rashely togither, and applyed for one purpose, but reson is the expressing of a iust matter with witty persuasiōs, furnished with lerned knowledge: suche knowledge had Moyses, beeyng experte in all learning of the Egyptians, as the Scriptures declare, and therefore was able to persuade the stubborne people of the Iewes, althoughe not without greate payne. Suche knowledge and suche reasons dydde Druys shewe, whiche was the first lawemaker of all the west part of Europe. Like reason and wisdome did Zamolxis vse amongest the Gothes: Lycurgus vnto the Lacedemonians, Zaleucus to the Locrians, Solon to the Atheniēses,

Drays was son to king Sarron, and succeded him in his kingdome.

ses, and Dunwallo Molmitius two thousād yeares past, amongest the old Britains of this Realme. And thereby came it to passe, that theyr lawes continued long, till more perfecte reason altered many of them, and wylfull power oppressed most of them.

At the beginning when these wyse menne perceyued how harde it was to bryng the rude people to vnderstande reason, they iudged the beste meanes to attayne thys honeste purpose to depende of learning in euery kynde, for by learning, as Ouide sayth: *Pectora mollescunt, asperitásque fugit*: Stoute stomackes do waxe mylde, and sharp fiercenesse is exiled. Therfore as Berosus doth testifie, Sarron, that was the thirde king ouer all this weste parte of Europe, for to bring the people from beastly rage to manly reason, did erecte Schooles of liberal artes, which took so good successe, that his name cōtinued in that sort famous aboue two thousand yeres after: for Diodorus Siculus, which was in the tyme of Julius Cesar, maketh mētion of the learned men of the Celtes, and nameth them Sarronides, that is to say, Sarron his Scholers and folowers.

Sarron died before Christes birthe 1913. yeares, after he had reigned. 48. yeares.

Among these artes that then were taught, some did informe the tongue, and make men

able

able both to vtter aptely their mynde, and also to perswade, as Grammer, Logike, and Rhetorike, althoughe not so curiously as in thys tyme: some other did appertayne to the iuste order of partition of Landes, the true vsyng of weyghtes, measures, and reckenings in all sortes of bargaynes, and for order of buylding and sundry other vses, those were Arithmetike and Geometrie. Agayne to incourage men to the honour of God, they taughte Astronomie, wherby the wonderfull workes of God were so manifestly set forth, that no mans tongue nor penne can in lyke sorte expresse his infinite power, his vnspeakable wysedom, and his exceeding goodnesse towarde man, wherby he doth bountifully prouide for man all necessaries, not only to liue, but also to liue pleasauntly. And so was their confidence in Gods prouidence strongly stayed, knowing his goodnesse too bee suche, that he woulde helpe man as he coulde, and his power to bee so greate, that he coulde do what he would: and thirdly his wisedome to be so pure, that he would doe nothing, but that that was beste. Beside these sciences they taught also Musike, whiche moste commonly they didde applye partely to Religious Seruices, to drawe men to delighte therein, and

partly

partely to Songes made of the manners of men, in prayse of Vertue, and discommendation of Vice, whereby it came to passe, that no man woulde displease them, nor doe anye thyng euyll that might come to their hearing: for their songs didde make euill men more abhorred in that tyme, than any excommunication dothe in this tyme. The posteritie of these Musitians contynewe yet bothe in Wales and Irelande, called Bardes vnto this daye, by the auncient name of Bardus, theyr fyrste founder.

And as these sciences did increase, so dydde vertue increase thereby. Agayne, as these sciences did decay, so vertue lost hir estimation, and consequentely was litle in vse: whereof to make a full declaration were a thing mete for a Prince to heare, but it woulde require a peculiare treatise. Wherefore at thys present I count it sufficient lightly to haue touched this matter in generall wordes, and to saye no more of the particularitie thereof, but onelye touching one of those sciences, that is Arithmetike, by whiche not onely iust partition of landes was made, but also touchyng buying and selling, al Assises, weights, and measures were deuised, and all reckenyngs and accounts

This Bard Druidius the fyfth kyng of the Celt reigned. 60 yeares, and died. 1831 yeares before Christ.

driuen, yea by proportion of it were the true orders of Iustice limitted, as Aristotle in his Ethikes doth declare, and the degrees of estates in the common wealthe established. Althoughe that proportion bee called Geometricall, and not Arithmeticall, yet doth that proportion appertayne to the Arte of Arithmeticke: and in Arithmetike is taught the Progression of suche proportions, and all thynges thereto belonging. Wherfore I may well saye, that seyng Arithmetike is so many ways needefull vnto the fyrste planting of a common wealth, it must needes bee as muche requyred to the preseruation of it also, for by the same meanes is any common wealth continued, by which it was erected and established. And if I shall in small matters in apperance, but in dede verie weightie, put one example or two, what shall wee saye for the Statutes of this Realme, whiche be the onely stay of good order in manner now? As touching the measuring of grounde by lengthe and bredth, there is a good and an auncient Statute made by arte of Arithmetike, and nowe it shalbe to litle vse, if by the same art it be not practised and tryed: for the assise of breade and drynke, the two most common and most necessary things

for sustentation of man, there was a goodly ordinance in the lawe made, which by ignorãce hath so growne out of knowledge and vse, that fewe men doe vnderstande it, and therfore the statute bookes wonderfully corrupted, and the commons cruelly oppressed: notwithstanding some men haue written, that it is to doubtfull a matter to execute those Assises by those Statutes, by reason they depende of the standerde of the coyne, whiche is much changed from the state of that tyme when those Statutes were made. Thus shall euery man reade (that listeth) in the Abridgement of statutes, in the title of Weightes and measures, in the seuenth number of the Englishe booke, where he shoulde haue translated a good ordinaunce, whiche is sette forthe in the Frenche booke: but no meruayle if the Abridgemente doth omitte it, seing the greate boke of Statutes doth omitte the same Statute, as it hath done diuers other very good lawes. And this is the fruict of ignoraunce, to reiecte and condemne all that it vnderstandeth not, although they vse some clokes for it: but suche clokes as beeing allowed, myght serue to repel al good Lawes, which God forbid.

Againe there is an auncient order for Assise

of

of fyrewood, and coales, whiche was renued not many yeares paste, and nowe howe Auarice and Ignorance doth canuas that statute, it is to pitifull to talke of, and more myserable to feele.

Furthermore for the statute of coinage, and the standerde thereof, yf the people vnderstode rightly the statute, they should not nor would not (as they often doe) gather an excuse for their follie thereby: but as I sayde, these statutes by wysedome and good knowledge of Arithmetike were made, and by the same muste they be continued. And let ignorance no more meddle with the vse of them, than it did with the making of them. Oh in howe miserable case is that Realme, where the ministers and interpreters of the Lawes are destitute of all good sciences, which be the keys of the Lawes. Howe can they eyther make good Lawes, or mayntayne them, that lacke that true knowledge wherby to iudge them. And happy maye that Realme be accounted, where the Prince hymselfe is studious of learnyng, and desireth to vnderstande equitie in all Lawes. Therfore moste happie are wee the louyng subiectes of your maiestie, whiche may see in youre highnesse not only such towardnesse, but also suche

knowledge of dyuers artes, as seldome hathe ben seene in any Prince of such yeares, wher=by wee are inforced, to conceyue this hope: Certaynly, that hee whiche in those yeares se=keth knowledge when knowledge is least e=stemed, and of suche an age can discern them to be enimies bothe to his royall person, and to his Realmes, whiche labour to withdrawe him from knowlege to excessiue pastime, and from reasonable study to idle or noysom plea=sures, he must needes when he cometh to more mature yeares, bee a moste prudent Prince, a most iuste Gouernoure, and a righte Iudge, not onely of his subiectes commonly, but also of the ministers of hys lawes, yea and of the lawes them selfe. And to be able to conceyue the true equity and exact understanding of all his lawes and statutes, to the comforte of his good subiectes, and the confusion and reprocke of them whiche laboure to obscure or peruerte the equitye of the same lawes and statutes. How some of those statutes maye be applyed to vse as well in this our time, as in anye o=ther tyme, I haue peculiarely declared in this booke, and some other I haue omitted for iust considerations till I may offer them firste vn=to your Maiestie, to waye them, as to your

highnesse shall seeme good: for many things in them are not to be published without your highnesse knowledge and approbation, namely bicause in them is declared all the rates of alloyes for all standerds from one unce upward, with other mysteries of mynte matters, and also moste parte of the varieties of coynes that haue bin currant in this your Maiesties realme by the space almost of .vi. hundred yeares laste past, and many of them that were currant in the tyme that the Romains ruled here.

All whiche with the auncient discription of England and Ireland, and my simple cẽsure of the same, I haue almost completed to be exhibited to your highnesse. In the meane season most humbly beseeching your Maiestie, to accept this simple treatise, not worthie to be presented to so high a Prince, but that my lowly request to your Maiestie is, that this amongst other of my bookes may passe vnder the protection of your highnes, whom I beseech God moste earnestly, and dayly, according to my duetie, to aduance in all honour and princely regalitie, and to increace in all knowledge, iustice, and godlie policie. Amen.

Your maiesties moste obedient subiect and seruaunt Robert Recorde.

To

TO THE LOuing Reader,

The Preface of M. Robert Recorde.

SORE OFTENTYmes haue I lamēted with my selfe the infortunate condition of England, seing so many greate Clerkes to aryse in sundrye other partes of the worlde, and so fewe to appeare in this oure nation: where as for pregnancie of naturall witte (I thynke) few nations doe excell English men. But I can not impute the cause to any other thyng than to the contempt or misregarde of learning. For as Englyshe men are inferior to no menne in mother witte, so they passe all men in vayne pleasures, to whych they maye attayne with great payne or labour: and are as slacke to any neuer so greate commoditie, if there hang of it any paynfull studie or trauelsome laboure.

Howbeit, yet all men are not of that sorte, though

though the moste parte be, the more pitie it is: but of them that are so gladde not onely with paynfull studie and studious paine to attayne learning, but also with as greate studie and paine to communicate their learning to other, and make all England, if it mought be, partakers of the same, the moste part are suche, that vnneth they can supporte their owne necessarie charges, so that they are not able to beare any charges in doing of that good, that else they desire to do.

But a greater cause of lamentation is this: that when learned menne haue taken paynes to doe thinges for the ayde of the vnlearned, scarce they shall be allowed for their well doing, but deryded and scorned, and so vtterly discouraged to take in hande any lyke enterpryse agayne. So that if any be found) as there are some) that doe fauour learnyng and learned wittes, and can bee contente to further knowledge, yea onely with their word, suche persons, though they be rare, yet shall they encourage learned menne to enterpryse some thyngs, at the leaste, that Englande maye reioyce of. And I haue good hope that England will (after shee hath taken some sure taste of learning, not onely bryng forth more fauourers

ters of it, but also suche learned men, that she shall be able to compare with any Realme in the worlde. But in the meane season, where so fewe regarders of learning are, how greatly they are to be esteemed that doe fauour and further it, my penne will not suffice at full to declare.

Therefore gentle Reader, where as I doe vppon moste iuste occasion iudge, yea and knowe assuredlye, that there be some men in this realme, which bothe loue and also muche desyre to further good learnyng, and yet am not well able to write their condigne prayse for the same, I thynke it better wyth sylence to ouerpasse it, than eyther to saye too lyttle of it, or to prouoke agaynst them the malice of suche other, whyche doe nothyng themselfe that is prayse worthye, and therefore can not abyde to heare the prayse of any other mannes good deed.

And consydering theyr greate fauour vnto learnyng, though I my selfe be not worthie to be reckened in the number of greatlerned menne, yet am I bolde to put my selfe in preace with suche abilitye as God hath lente mee, thoughe not with so great cunnyng as many menne, yet with as great affection as

any man, to healpe my countreymenne, and will not ceasse daylye, (as muche as my small abilitie will suffer mee) to endite some suche thyng, that shall bee to the instruction, though not of learned menne, yet at the leaste of the vulgare sorte, whose argument allways shall be suche, that it shall delyte all learned wittes, though they doe not learne any greate things out of it.

But to speake of this present booke of Arithmetike, I dare not nor will not sette it forthe with anye wordes, but remit it to the iudgement of all gentle readers, and namely suche as loue good learning, beseechyng them so to esteme it, as it doth seme worthie. And so eyther to accept the thing for it selfe, eyther at the least to allowe my good endeuor. But I perceyue I neede not vse anye persuasions vnto them, whose gentle nature and fauourable mynde is readye to receyue thankfully, and interprete to the beste all suche enterpryses attempted for so good an ende, though the thyng doo not alwayes satisfie mennes expectation. Thys considered, dydde bolden me to publishe abroade thys little booke of the Arte of numbryng, whyche yf you shall receyue fauourablye, you shall encourage mee to gratifie

gratify you hereafter with some greater thing.

And as I iudge some men of so louyng a mynd to their natiue countrey that they wold muche reioyce to see it to prosper in good learning and wittie Artes, so I hope well of all the reste of Englishemen, that they wil not be vnmyndefull of hys due prayse, by whose meanes they are healped and furthered in anye thing. Neyther oughte they to esteeme this thing of so little value, as many menne of lyttle discretion oftentymes doe: For who so setteth small price by the wittie deuyse and knowledge of numbering, hee little consyderetl it to bee the chiefe poynte, (in maunner) wherby men differ from all brute beastes: for as in all other thynges (almoste) beastes are partakers with vs, so in numbering we differ cleane from them, and in manner peculiarly, sith that in manye thynges they excell vs agayne.

The Foxe in craftie witte exceedeth most men,
A Dogge in smelling hath no man his peere,
To foresight of weather if you looke then,
Many beastes excell man, this is cleere.
The wittinesse of Elephants doth letters attaine
But what cunning doth there in the Bee remain?

The Emmet forseeing the hardnesse of winter,
Prouideth vitayles in tyme of Sommer.
The Nightingale, the Linet, the thrush, the lark.
In Musicall harmonie passe many a Clarke.
The Hedgehogge of Astronomie semeth to know
And stoppeth his caue wher the wind doth blow
The Spider in weauing such arte doth showe,
No man can him mende, nor follow I trowe.
VVhen a house will fall, the Myse right quick,
Flee thence before, can man do the like?

Manye thynges else of the wittynesse of beastes and byrdes myght I heare saye, saue that an other tyme of them I intend to write wherein they excell in manner all men, as it is dayly seene: but in number was there neuer beast found so cunning, that could knowe or discern one thing from many: as by dayly experience you may well consyder, when a Bytche hathe manye whelpes, or a Henne manye Chyckens: and lykewayes of other what soeuer they bee, take from them all their young, sauyng onely one, and you shall perceyue playnelye, that they mysse none, though they will resiste you in takyng them away, and will seke them agayn if they maye know

knowe where they bee, but else they will neuer misse them truely, but take awaye that one that is lefte, and then will they crye and complayne: and restore to them that one, then are they pleased agayne: so that of number this may I iustly saye, It is the onely thyng (almost) that separateth man from beastes. Hee therefore that shall contemne number, he declareth hym selfe as brutishe as a beaste, and vnworthie to be counted in the felowshippe of men. But I truste there is no man so foule overseene, though many ryghte smally doe it regarde.

Therfore wil I now stay to write against suche, and returne agayne to this my Booke, whiche I haue wrytten in the forme of a Dialogue, bicause I iudge that to be the eaysest way of instruction, when the Scholer maye aske euery doubt orderly, and the maister may answere to his question playnly.

Howebeit I thinke not the contrarie, but as it is easyer to blame an other mans worke than to make the lyke, so there will bee some that will fynde faulte, bycause I write in a Dialogue: but as I coniecture, those shal be suche, as doe not, cannot, eyther will not perceyue the reason of right teachyng, and therefore

fore are vnmete to be answered vnto, for suche men with no reason will be satisfyed.

And if any man obiecte that other bookes haue beene written of Arithmetike already so sufficiently, ŷ I needed not nowe to put pen to the booke, except I wil condemne other mens writyngs: to them I answere. That as I condemne no mans diligence, so I know that no one man can satisfie euery man: and therfore lyke as many doe esteeme greately other bookes, so I doubt not but some will like this my Booke aboue anye other Englyshe Arithmetyke hytherto wrytten, and namely suche as shall lacke instructers, for whose sake I haue so playnely set forth the examples as no booke (that I haue seene) hath done hytherto: whych thing shall be great ease to the rude readers.

Therfore gentle reader, thoughe this booke can be but small ayde to the learned sorte, yet vnto the simple ignorant (which needeth most helpe) it maye bee a good furtheraunce and meane vnto knowledge.

And thoughe vnto the Kyng his Maiestie priuately I do it dedicate, yet I doubt not such is his clementie) but that hee can be contente, yea and muche desirous, that all his louyng

subiectes

subiectes shall take the vse of it, and employe the same to their most profite: Which thyng if I perceyue that they thankfully doe, and receyue with as good wyll as it was written, then will I shortely with no lesse kyndnesse sette forth such introductions into Geometrie and Cosmographie, as I haue at times promised, and as hitherto in Englishe hath not bin enterprised, wherwith I dare saye all honeste heartes will be pleased, and all studious wits greatly delighted.

I will saye no more, but lette euery manne iudge as he shall see cause. And thus for thys tyme I will stay my penne, committyng you all to that true fountayne of perfecte number, whiche wrought the whole worlde by Number and measure: he is Trinitie in Unitie, and Unitie in Trinitie: To whom be all prayse, honour and glorie.

AMEN.

The contentes of the firste Dialogue.

The declaration of the profite of Arithmetike.
Numeration with an easy and large Table.

Addition, Subtraction. Multiplication, Diuision,	with diuers examples, and som newe formes of working.

Reduction with diuers declaratiōs of coynes, weightes, and measures of sundry formed now newly added to these other rules.
Progression bothe Arithmetical and Geometricall, with certayne questions touchyng the same.
The Golden rule, and the backer rule, with diuers questions thereto belonging.
The double rule of proportion.
The rule of Felowship both with Time, and without Tyme.
Unto all these is added their profe.

The seconde Dialogue.

The first. v. kinds of Arithmetike wrought by Counters.
The common kindes of casting accompts after Marchantes fashion and auditors also.
Numbryng by the hande, newly added.

The

The contentes of the second part *touching fractions.*

Before the Introduction of Arithmetike, these Figures must be learned.

	1	2	3	4	5	6	7	8	9
Figures of Number.	j	ij	iij	iiij	v	vj	vij	viij	ix

1 0	x.
2 0	xx.
3 0	xxx.
4 0	xl.
5 0	l.
6 0	lx.
7 0	lxx
8 0	lxxx.
9 0	xc.
100.	c.

CC. two hundred. &c.
D. fiue hundred. &c.
D.C. sixe hundred. &c.
M. a thousand. &c.

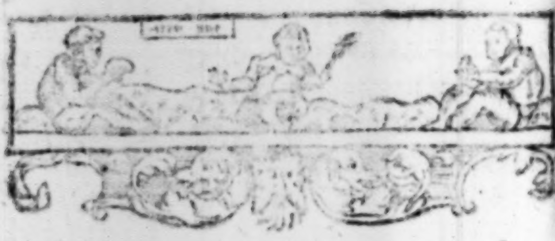
M. D. LXXV.

Figures of Money.

c, a cee, the.xvj.
q, a kewe the.viij. } part of a penny.
q̄, a farthing, the.iiij.

ob. an halfe pennie.
j d, a pennie.
j s, a shilling.
j li, a pounde.

Thou (ô God) haste ordered all things in Measure, Number, and weyght.



A DIALOGVE BETWEENE THE Mayster and the Scholer, teaching the arte and vse of Arithmetike with the penne.

THE SCHOLER SPEAKETH.

SYR, such is youre authoritye in myne estimation, that I am content to consent to your saying, and to receyue it as truth, though I see none other reason yͤ dothe leade mee therunto: whereas els in mine owne cõceit it appeareth but vaine, to bestowe anye tyme priuatelye in learning of that thing, that euery childe maye and dothe learne at all tymes and houres, when hee dothe any thing him selfe alone, and much more when hee talketh or reasoneth with other.

MAYSTER. Lo this is the fashion and chaunce of all them that seeke to defende theire

blinde ignorance: that when they thinke they haue made strong reason for themselues, then haue they proued the quyte contrarye. For yf numbring bee so common (as you graunte it to bee) that no man can doe any thing alone, and much lesse talke or bargayne with other, but hee shall still haue to doe with Number: this proueth not Number to bee contemptible and vile, but rather right excellent and of high reputation, sith it is the groūde of all mens affayres, so that without it no tale can bee tolde, no communication without it can bee long continued, no bargaining without it can duely bee ended, nor no businesse that man hathe, iustly completed. These commodities (if there were none other) are sufficient to approue the worthinesse of Number. But there are other vnnumerable farre passing all these, which declare Number to exceede all praise. Wherefore, in all greate workes are Clerkes so much desired? Wherfore are Auditours so richly feed? What causeth Geometritrians so highly to be enhaunced? Why are Astronomers so greatly aduaunced? Bycause that by Number such things they doe finde, which elles shoulde farre excell mans mynde.

Scholer. Verily Syr if it be so, that these

men

men, by Numbring their cunning do attaine, at whose great woorkes most men do wonder, then I see well I was much deceyued, and Numbring is a more cunning thing than I tooke it to bee.

Mayster. If Number were so vile a thing as you did esteeme it, then needeth it not to be vsed so muche in mens communication. Exclude Number and answere mee to this question. How many yeares olde are you?

Scholer. Mum.

Mayster. How many dayes in a weeke? how many weekes in a yeare? What landes hath your father? How many men doth he keepe? Howe long is it sithe you came from him to mee?

Scholer. Mum.

Mayster. So that if Number want, you answere all by Mummes: Howe many myle to London?

Scholer. A poke full of Plummes.

Mayster. Why, thus may you see, what rule Number beareth, and that if Number bee lacking it maketh men dumbe, so that to moste questions they must answere Mum.

Scholer. This is the cause sir, that I iudged it so vyle, bycause it is so common in tal-

king euery while: For plentie is not deintie, as the common saying is.

Mayster. No, nor Store is no sore: perceyue you this? The more common that a thing is, being needfully required, the better is the thing, and the more to be desired. But in Numbring as some of it is light and plaine, so the moste part is difficult, and not easie to atteine. The easier part serueth all men in common, and the other part requireth some learning. Wherefore as without Numbring a man can do almost nothing, so with the helpe of it, you may attaine to all thing.

Scholer. Yea sir? Why? then it were best to learne the arte of Numbering first of all other learning, and then a man neede learne no more, if all other come with it.

Mayster. Nay not so: but if it be first learned, then shall a manne be able (I meane) to learne, perceyue, and attayne to other sciences, which without it, he should neuer get.

Scholer. I perceyue by your former words, that Astronomie and Geometrie depende muche of the helpe of Numbering, but that other sciences, as Musicke, Phisicke, Law, and Grammar and such like, haue any helpe of Arithmetike, I perceyue not.

Mayster.

Mayster. I may perceyue your great clerk-linesse by the ordering of your sciences: but I will let that passe nowe, bicause it toucheth not the matter that I intend, and I will shew you how Arithmetike doeth profite in all these, somewhat grossely, according to your small vnderstanding, omitting other reasons more substantiall.

First (as you recken them) Musicke hathe not onely great helpe of Arithmetike, but is made & hath his perfectnesse of it: for al Musike standeth by Number and Proportion. Musicke

And in Phisicke, beside the calculation of Criticall dayes, with other things whiche I omitte, howe can anye man iudge the Pulse rightlye, that is ignorant of the proportion of Numbers? Phisicke.

And as for the Lawe, it is plaine, that the man that is ignorant of Arithmetike, is neyther meete to be a Iudge, neyther an Aduocate, nor yet a Proctour. For howe can he well vnderstande an other mans cause appertey-ning to distribution of goodes, or other dettes, or of summes of money, if hee bee ignorant of Arithmetike? This oftentymes causeth right to be hindered, when the Iudge either deliteth not to heare of a matter that hee perceyueth The lawe.

not, either can not iudge it for lacke of vnderstanding: This commeth by the ignorance of Arithmetike.

Grammer.

Nowe as for Grammer, me thinketh you should not doubt in what it needeth Number, sith you haue learned the Nounes of all sortes, Pronounes, Verbes, and Participles, are distinct diuersly by Numbers: besides the varietie of Nounes of Number, & Aduerbes. And if you take away Number from Grammer, then is all the quantitie of Sillables lost. And many other wayes doth Number help Grammer. Whereby were all kindes of meters foūd and made? Was it not by Number?

Philosophy

But howe needefull Arithmetike is to all partes of Philosophie, they may soone see, that do reade either Aristotle, Plato, or any other Philosophers writing. For all their examples almoste, and their probations depend of Arithmetike. It is the saying of Aristotle, that hee that is ignorant of Arithmetike, is meete for no science. And Plato his Mayster wrote a like sentence ouer his Schoolehouse doore, *Let none enter in hither* (quoth hee) *that is ignorante of Geometrie.* Seeing hee woulde haue all his Scholers experte in Geometrie, much rather hee woulde the same in Arithmetike, without

without which Geometry can not stande.

And how needefull Arithmetike is to Diui- Diuinitie.
nity, it appeareth, seeing so many Doctors gather so greate mysteries out of Number, and so much doe write of it. And yf I shoulde go about to write all the commodities of Arithmetike in ciuill actes, as in gouernaunce of common weales in tyme of peace, and in due prouision and order of armyes in tyme of warre: For numbring of the hoste, summing of their wages, prouisions of vitayles, vewing of artillerye, with other armoure: Beside the cunningest poynt of all, for casting of grounde for encamping of men, with such other like. And howe many wayes also Arithmetike is conducible for all pryuate weales, of Lordes and all possessioners, of marchauntes, and all other occupyers, and generally, for all estates of men, besides Auditors, Treasorers, receyuers, stewardes, bayliffes, and such like, whose offices without Arithmetike is nothing. If I shoulde (I say) particularlye repeate all such commodities of this noble science of Arithmetike, it were ynough to make a very greate booke.

Scholer. No, no syr, you shall not neede: For I doubt not, but this that you haue sayde,

were ynough to perswade any man to thinke this arte to be right excellent and good, and so necessary for man, that (as I thinke nowe) so much as a man lacketh of it, so much he lacketh of his sense and witte.

Mayster. What? are you so farre chaunged since, by hearing the few commodities in generall? By likelyhode you would be far chāged, if you knewe all ẏ cōmodities particular.

Scholer. I beseech you sir, reserue those commodities that rest yet behinde, vnto their place more conuenient. And if yee will bee so good as to vtter at this tyme this excellent treasure, so that I may bee somewhat enriched thereby, and if euer I shall be able, I will requite your payne.

Mayster. I am very glad of your request, and I will doe it speedely, sith that to learne it you bee so ready.

The duty of a scholer

Scholer. And I to your authoritye my witte doe subdue, whatsoeuer you say, I take it for true.

Mayster. That is to much, and meete for no man, to bee beleued in all things, without shewing a reason. Though I might of my Scholer some credence require, yet except I shew reason, I doe it not desire. But nowe sith

you

you are so earnestlye set this arte to attayne, best it is to omitte no tyme, leaste some other passion coole this greate heate, and then you leue of before you see the ende.

Scholer. Though many there bee so vnconstant of minde, that flitter and turne with euery winde, which often beginne, and neuer come to the ende, I am none of theire sorte, as I trust you partly know. For by my good will what I once beginne, tyll I haue it fully ended, I would neuer blyn.

Mayster. So haue I founde you hitherto in deede, and I truste you will increase rather than go backe. For better it were neuer to assaye, than to shrinke and flee in the middle way. But I truste you will not so doe, therefore tell me briefly, What call you the science that you desyre so greately?

Scholer. Why syr? you know.

Mayster. That maketh no matter, I woulde heare whether you knowe, and therefore I aske you. For greate rebuke it were, to haue studyed a science, & yet can not tell howe it is named,

Scholer. Some call it Arithmetike, & some Augrime. Arithmetike.

Maister. And what dothe those names

 betoken?

betoken.

Scholer. That, if it pleaſe you, of you woulde I learne.

Mayſter. Bothe names are corruptlye
Ἀριθμη- written, Arſmetrike for Arithmetike, as the
τική. Greekes call it, and Augrim for Algoriſme, as the Arabians ſounde it, which bothe betoken
Ἀριθμός. the ſcience of numbring. For Arithmos in Greeke, is called Number: and of it commeth Arithmetike, the arte of Numbring. So that Arithmetike is a ſcience or arte teaching the manner and vſe of numbring. This arte may be wrought diuerſly, with Penne or wͭ Counters. But I will firſte ſhew you the working with the penne, and then the other in order.

Scholer. This I will remember. But how many things are to bee learned, to attayne this arte fully?

Mayſter. There are reckened commonlye vij. partes or workes of it.

Numeration, Addition, Subtraction, Multiplication, Diuiſion, Progreſſion, and Extraction of rootes: to theſe ſome men adde Duplation, Triplation, and Mediation. But as for theſe laſt three, they are contained vnder the other ſeuen. For Duplation and Triplation, are cõtained vnder Multiplication, as it ſhall appeare

appeare in their place. And mediation is contayned vnder Diuision, as I will declare in his place also.

Scholer. Yet then there remayne the first seuen kindes of numbring.

Mayster. So there dothe: Howbeeit, If I shall speake exactly of partes of Numbring, I must make but fiue of them: For Progression is a compounde operation of Addition, Multiplication and Diuision. And so is the extraction of Rootes. But it is no harme to name them as kindes seuerall, seeing they appeare to haue some seuerall working. For it forceth not so much to contende for the Number of them, as for the due knowledge and practising of them.

Scholer. Then you will that I shall name them as seuen kindes distinct. But now I desire you to instruct me in ÿ vse of ech of them.

Mayster. So will I, but it must bee done in order: for you may not learne the last as soone as the first: but you must learne them in that order, as I did reherse them, if you will learne them spedily and well.

Scholer. Euen as you please. Then to begin, Numeration is the first in order: what shall I do with it?

Mayster.

Mayster. First you muste knowe what the thing is, and then after learne the vse of the same.

NVMERATION.

NVMERATION, is that Arithmeticall skill, whereby we may duely value, expresse and reade any Number or summe propounded: or else in apte figures and places, sette downe any Number knowne or named.

Scholer. Why? then me thinketh you put a difference betwene the value and the figures?

Mayster. Yea so doe I: For the value is one thing, and the figures are another thing: and that commeth partly by the dyuersitie of figures, but chiefly of the places wherein they be set.

Scholer. Then I muste knowe here three things: the Value, the Figure, and the Place.

Mayster. Euen so: but yet adde Order to them as the fourth. And first marke, that there are but ten Figures, that are vsed in Arithmetike: and of those ten, one doth signifie nothing, which is made like an o, and is called

priuately

priuately a Cyphar, though all the other som- time be likewise named. The other nyne are called Signifying figures, and be thus figured. A Ciphar. Figures.

1 2 3 4 5 6 7 8 9

And this is their value.

j. ij. iij. iiij. v. vj. vij. viij. ix.

But here muste you marke, that euery Figure hathe two values: One alwayes certaine, that it signifieth proprely, which it hath of his Forme: and ỹ other uncertayne, which he taketh of his Place.

A Place is called the seate or roome that a Figure standeth in. And looke how many figures are written in one summe, so many places hathe that whole number. And the firste place must bee called that, that is nexte to the right hande, and so reckening by order toward the left hande, so that that place is laste, that is next to the left hande, as for example: If there stoode before you sixe men in a rowe, side by side, and you shoulde tell them as they stand in order, beginning with the man that were next to your righte hande: then hee that were next him shoulde bee called the seconde, and so forthe to the farthest from your right hande, A place.

which

which is the first and the last.

Scholer. Sir I perceyue you well : so might I recken letters or any other thing. As if I shoulde write.viij. letters after this order, a, b, c, d, e, f, g, h, nowe must I say : h, is the first, g the ii, f the iij, e the iiij, d the v, c the vj, b the seuenth, and a the eight.

Mayster. That is well done. And after the same sort vse hereafter, that what I declare by one example, doe you expresse by an other, and so I shall perceyue whither you vnderstande it or no. And so passe ouer nothing, till you perceyue it well, and be expert therein.

Scholer. Sir, I pray you howe many of these places be there in all ?

Mayster. There is no certaine number of them, but there are somtimes more & somtimes fewer, according to the sum that is expressed. For so many as the figures are, so many are ẏ places : and the last place is so called, not bycause it is last of all other, but it is the last of that present summe, and it may bee the middle place in an other summe.

Scholer. Me seemeth I perceyue this very well, as touching the order of reckening of the places: But as for the number of them, you saye there is no certaintie. Nowe there resteth

to declare the valew of the figures by diuersity of places, which you called, The valewe vncertaine.

Mayster. But firste let mee heare whether you know perfectly the certayne valewe. Valevve vncertaine.

Scholer. Yes syr as you wrote them, so I marked them.

Mayster. How write you then fiue?

Scholer. By this figure. 5.

Mayster. And how sixe?

Scholer. Thus. 6.

Mayster Write these three numbers eche by it selfe as I speake them. vij. iiij. iij.

Scholer. 7. 4. 3.

Mayster. How write you these four other, ij, j, ix, viij?

Scholer. Thus (I trowe) 2, 1, 6, 8.

Mayster. Nay, there you misse: Looke one myne example agayne.

Scholer. Syr, troth it is, I was to blame, I tooke 6 for 9, but I will bee warer hereafter.

Mayster. Nowe then take heere, these certayne valewes euery figure representeth, when it is alone written without other figures ioyned to him. And also when it is in the firste place, though many other doe followe: as for example: This figure 9 is ix. standing nowe alone,

alone.

Scholer. How? is hee alone, and standeth in the middle of so many letters?

Maister. The letters are none of his felowes. For if you were in Fraunce in ye middle of a M. Frenche men, if there were none Englishe man with you, you woulde recken your selfe to be alone.

Scholer. So it is. Then 9 without more figures of Arithmetike, betokeneth. ix. whatsoeuer other letters be about it.

Mayster. Euen so, and so doth it, if it be in the firste place ioyned with other, howe many soeuer we followe, as in this example, 3679, you see 9 in the firste place, and dothe betoken nyne, as if hee were alone.

Scholer. I perceyue that. And dothe not 7 that standeth in the seconde place, betoken vij? and 6 in the thirde place, betoken vj? And so 3 in the fourth place, betoken three?

Maister. Their places bee as you haue saide, but their valewes are not so. For as in ye firste place euerye figure betokeneth his owne valewe certayne onely, so in the second place euery figure betokeneth his owne valewe certaine x. tymes: as in the example, 7 in the seconde place is vij. times x. that is, lxx. And in the

the thirde place, euery Figure betokeneth his owne valewe a hundreth tymes, so that 6 in ẏ place betokeneth vj.C. And in the iiij.place, euerye Figure betokeneth his owne valewe a M.times, as in the foresayde number 3. in the fourthe place, standeth for iij.M. And in the fifth place, euery figure standeth for his owne valew.x.M.tymes. And in the vj.place, a C.M. tymes. And in the vij.place, a M.M.tymes. And in the viij.place, x.M.M.tymes: so that euery place exceedeth the former. x.tymes,

Sc. As thus: if I make this Number at all aduentures, 91359684, here are viij.places. In the firste place is 4, it betokeneth but foure: in the seconde place is 8, and betokeneth.x. tymes eyght, that is, lxxx. In the thirde place is 6, and betokeneth, vj.C. In the fourth place 9, is ix.M. And 5 in the v.place, is x.M.times 5, that is, l.M. So 3 in ẏ sixt place, is a C.M. times 3 that is CCC.M. Then 1 in the vij. place, a M.M. And 9 in ẏ viij.place, x M.M. tymes 9, that is, xc.M.M. But now I canne not easily nor quickly reade it in order. — A generall rule,

Mayster. That shall you practise by this meanes. First put a pricke ouer the fourthe Figure, and so ouer the vij. And (if you haue so manye) ouer the x, xiij, xvj. and so forthe, still

 leauing

leauing two figures betwéene eche two prickes. And those two roomes betwéene the prickes, are called Ternaries.

Ternarie. Trinitie. Then begin at the laste pricke, and sée how many figures are betwéene him and the ende, which can not passe thrée, reckening himselfe for one: then pronounce them as if they were written alone from the reste, and adde at the ende of their valew so many tymes thousand, as your number hath prickes.

After that come to the next thrée figures, & sounde them as if they were a parte from the reste, and adde to their valewe so many tymes thousandes, as there are prickes betwene them and the first place of your whole number. And so doe by euery other. iij. figures following, if you haue mo. As in example, 91359684. this was your number.

Put a pricke ouer 9 in the fourth place, and ouer one in the vij. place, and then no more, (for youre places come not to tenne) as thus:

9135968 4.

Now go to the laste pricke ouer 1, and take it and the figure 9 that followeth it, and valew them alone.

Scholer. 91 that is xcj.

Mayster. So is it: but then adde for the number

number of your prickes twise M.

Scholer. That is xcj. thousand thousande.

Mayster. So is it. Then take the three other figures from one to the next pricke, and value them.

Scholer. 359. that is CCC.lix.

Mayster. Now adde for the one pricke, that is betweene them and the first place, M.

Scholer. CCC.lix. thousand.

Mayster Then come to the other three figures that remaine.

Scholer. 684. that is vj.C.lxxxiiij.

Mayster. Nowe haue you valued all. And at the end of the last number you shal adde nothing, bicause there remaineth no pricke nor number after it: yet proue in an other number, as thus. 23086408910 5340.

Scholer. 23086408910 5340. I haue pricked them as you taught mee: but I am in doubt, whether I haue done well or no, because of the Cyphers: For I remember, you tolde mee that they doe signifie nothing, and therefore I doubt whether I shoulde recken them for a figure in setting of the prickes: and againe I knowe not wherefore they serue.

Mayster. That will I tel you now. In deed they

The vse of Cyphers.

they are of no value themselfe, but they serue to make vp number of places, and so maketh the figure following them, to bee in a further place, and therefore to signifie the more value: as in this example, 90, the Cipher is of no value, but yet hee occupieth the firste place, and causeth 9 to be in the seconde place, and so to signifie x. times 9, that is xc. so that two Cyphers thrusteth the figure following them, into the thirde place, and so forth.

Sc. Then I perceyue in the example aboue, I haue pricked well ynough: for though that Cypher that is pricked signifie nothing, yet must he haue the pricke, bicause he came in the xiij. place Then will I proue to number that summe. First there is 30 M.M.M.M. and then followeth 864 M.M.M. And what shal I nowe do? There is a Cypher in the thirde place, and no figure after him, but they that I haue reckened.

Ma. Hee did serue for them that you haue alreadie reckened, to make them in a place further than they should be if he were away: and therefore nowe you shall let him go. And so do always when he occupieth that place, next before any prick, which is the last of that Ternarie, and a cypher in the last place doth nothing

Sc. Then shall I say but 89 thousande M.

Mayster. So, but go forth.

Scholer. 105 thousand. Nowe are all my prickes spent, and yet remaine 340, so that I must value them CCC.xl onely.

Mayster. Nowe can you recken after this sort: and remember that every suche roome so parted, is called a Ternarie or Trinitie. Trinitie.

Some doe part suche great numbers with letters, after this maner.

a b a c b a c b a a b a c b a
2 3 0 8 6 4 0 8 9 1 0 5 3 4 0. In which example yee may see, y^e a supplyeth the roume of your pricke. And some doe parte the numbers with lines after this forme.

230|864|089|105|340. where you see as many lynes as you made prickes, & to one intent, save that the lynes doe more playnlie parte every three figures, according as they should be valued under one denomination.

Scholer. Yea sir, but if you should shew me a number so parted, I should take it for many numbers, and not for one.

Ma. So might you doe, not knowing my meaning. But what if I did set forth the nūber without lynes, and your selfe (for the ease of reckening) did so part it with lynes, woulde

you forget wherefore ye did it, and then take them for many numbers?

Scholer. No I trow not, but yet I doubt.

Mayster. Then use that that you like best, for all the three wayes are to one intent, saue (as I sayd) that the lines doe more plainly distinct the Denominations.

Denominations.

Scholer. What call you Denominations?

Mayster. It is the last value or name added to any summe. As when I say: CCxxij. poundes: poundes is the Denomination. And likewise in saying: 25 men, men is the Denomination, and so of other. But in this place (that I spake of before) the last number of euery Ternary, is the denomination of it. As of the first ternarye, the denomination is vnities, and of the seconde ternarye, the denomination is thousandes: and of the thirde ternarie, thousande thousandes, or millions: of the iiij. thousand thousand thousands, or thousand Millions: and so forth.

Scholer. And what shall I call the value of ẏ iij. figures that may be pronounced before the denominators? as in saying: 203000000, that is CCiij. millions. I perceyue by your wordes that millions is the Denomination: but what shall I call the CC iij. ioyned before the Millions?

Mayster.

Mayster. That is called the Numeratoꝛ oꝛ valewer, & the whole summe that resulteth of them bothe, is called the Summe, valewe oꝛ number. Numerators. Valevver. Summe.

Schol. Now, is there anye thing else to be learned in Numeration? oꝛ else haue I learned it fully?

Ma. I might here shewe you who were the first Inuentoꝛs of this arte, and the reasons of all these things that I haue taught you, but that will I reserue till yee haue learned ouer all the pꝛactise of this arte, least I should trouble your witte with ouer many things at the firste.

But yet this must you marke, that there are thꝛee kindes of number: one called digits: an other articles: and the thirde mixte numbers. Three kindes of number.

A Digit is any number vnder 10, as this: Digites.

1, 2, 3, 4, 5, 6, 7, 8, 9.

And 10 with all other that may bee diuided into x partes iuste, and nothing remaine, are called Articles: such are 10, 20, 30, 40, 50, &c. 100, 200, &c. 1000. &c. Articles.

And that number is called mixt, that containeth articles, oꝛ at the leaste one article and a digitte: as 12, 16, 19, 21, 38, 107, 1005, and so foꝛth. And foꝛ the moꝛe ease of vnderstanding Mixte.

remembrance, marke this: The diget number is neuer written with more thā one figure, but the article and the mixte number are euer written with more than one figure. And thus they differ, that the article hath euermore this ciphre o, in the firste place: and the mixt number hath euer there some diget.

Scholer. By these laste wordes, I perceyue it much better than I did before, and now (I thinke) I will neuer misse to know those three a sunder.

Maister. If you remember nowe all that I haue sayde, you haue learned sufficientlye this firste kinde of Arithmetike, called Numeration. Howbeit, I will yet exhorte you now, to remember bothe this that I haue sayde, and all that I shall saye, and to exercise your selfe in the practise of it: For rules wout practise, are but a lyght knowledge: and practise it is, that maketh men perfect and prompt in all things.

...seme...eth may...ry.

And as you haue learned to gather and expresse the value of a summe propounded, and set downe before you: so muste you practise to marke, note, or write downe, with apt figures, and in due places, any number, onely named or recited to you, or of your selfe imagined: as for a proofe: Howe note you, or write downe

this

this summe, Fiue thousande, two hundreth, fifty and seuen.

Scholer. This troubleth mee nowe, whether I shoulde beginne at the firste figure or at the laste. For reason (me thinketh) shoulde cause me to beginne at the firste: and yet if I write it as you spake it, I must beginne at the laste.

Mayster. When you knowe youre places perfectly, you may beginne where you list. But the more ease for your hand is to beginne with the laste, that is to say, as I did speake them. Yet for the more suerty, a while you maye beginne with the firste, repeating my wordes backewarde, thus: Seuen, fyfty, two hundred, fyue thousande: or else sounding them all by their diget or valewer: as thus: Seuen, fyue, two, fyue: for that waye is easiest. But then must you looke well, whether there bee any Cipher in your summe, that hee may bee sette in his place. As if your laste valewer of youre summe (as you speake it) bee aboue 9, then is there a Cypher in the firste place. And if it bee a hundred or aboue, then is there twoo Cyphers, one in the firste place, and an other in the seconde, and so forth.

But bicause this thing is suche that cā not bee seth forth without manẏ wordes, I thinke best here now at the ende of numeration, to adde a table easie, and ready for the first exercise of it.

Lo, this is the table.

The right side or hande.

11	10	9	8	7	6	5	4	3	2	1	
X. M. of milliōs.	M. of millions.	C. of millions.	X. of millions.	C. of thousādes.	Millions.	X. Thousandes.	Thousandes.	Hundredes.	Tennes.	Unities.	The denominatours of the place or valew vncertain
9	9	9	9	9	9	9	9	9	9	9	Nine.
8	8	8	8	8	8	8	8	8	8	8	Eyght.
7	7	7	7	7	7	7	7	7	7	7	Seuen.
6	6	6	6	6	6	6	6	6	6	6	Sixe.
5	5	5	5	5	5	5	5	5	5	5	Fiue.
4	4	4	4	4	4	4	4	4	4	4	Foure.
3	3	3	3	3	3	3	3	3	3	3	Three.
2	2	2	2	2	2	2	2	2	2	2	Two.
1	1	1	1	1	1	1	1	1	1	1	One.
0	0	0	0	0	0	0	0	0	0	0	Ciphar.
Eleuenth.	Tenth.	Ninth.	Eighth.	Seuenth.	Sixte.	Fifte.	Fourthe.	Thirde.	Seconde.	Firste.	The order of the places.

The names of the digits, valewes certaine, or valewers.

The lefte side or hande.

This

This table (as you maye see) hath eleuen places, and in eche of them are sette all the digites, whose certaine valne is written on the right hande of the table, and the value vncertaine on the left hande. So that by this table you may learne both howe to expresse any nu̅ber that you list, (if that it exceede not eleuen places) that is to say, lxxx. thousande millions, and so may you by the helpe of it, value all summes proposed vnder the sayde number.

For example: take the summe that I proposed before, whiche was fiue thousande, two hundred, fiftie and seuen. And if you will expresse it, take the first number (as I speake it) whiche is fiue M. whose valuer or certayne valne is v. and his vncertaine value or denomination is M. First you shall seeke at the right hande of the valuer. v. Then seeke along vnder the tytle of denomination towarde the left hande, till you finde thousandes, and vnder it right at the foote of the Table, is the number of the place, that is the fourth, wherein you must write your diget or valuer, fiue.

Afterward come to the second part of the nu̅ber, two hundred, whose valuer is. 2. and his denomination C. Seeke two at the right hand of

of the table, and go along vnder the denomi-nations, towarde the left hande, till you come vnder C: then looke to the foote of the table, & there shal you see the number of the place, that is to say, iij, wherein you must sette your digette.ij.

Then doe so by youre other two numbers that remaine, and you shall finde fiue in the seconde place for your fiftie, and 7 in the firste place for your seuen. And thus maye you doe with other numbers.

Scholer. Mayster I thanke you hartilye. I perceyue you seeke to instructe mee moste plainely and briefly, & not to hide your knowledge with subtile wordes as many do. For this rule is so playne, that I can desire it no playner. And though it seeme somewhat long, yet I perceyue it to be a sure way.

Mayster. So is it, and though it bee long, yet it is neyther too long, neyther too plaine for yong learners that lacke practise: for this table is in steade of a teacher, to them that lacke one. But now I trust I haue sayd ynough of Numeration: which after you haue well practised, then may you learne forth.

Why nō-rs are ced bak-varde.

Scholer. Yet I pray you in one thing to tell mee your iudgement. Why doe men recken

ken the order of the places backwarde, from the right hand to the left?

Mayster. In that thing all men do agree, that the Chaldeyes, which first inuented thys Arte, did set these figures as they set all theyr letters: for they wryte backewarde as you terme it, and so do they reade. And that may appeare in all Hebrue, Chaldeye, and Arabyke bookes, for they be not onely written from the right hande to the lefte, and so must bee read, but also the right ende of the booke is the beginning of it: where as the Greekes, Latines, and all nations of Europe, do write and reade from the lefte hande towarde the right: And all their bookes begin at the left side.

Scholer. That reason doth satisfie me.

Mayster. It neither satisfieth mee, neither liketh me well, bycause I see that the Chaldayes and Hebrues doe not so vse theyr owne numbers, as at an other time I will declare. But this plaine reason may best satisfie you presently: That seeing in pronouncing of numbers we keepe the order of oure owne reading, from the left hande to the right: And agayne, wee doe euer name the greater numbers before the smaller: it was reason, that the lesser places conteyning the lesser numbers, shoulde

ſhould be ſet on the right hand, and the greater places conteyning the greater numbers, to proceede towarde the left hande.

Scholer. This reaſon to me is ſo playne, that it ſeemeth now agaynſt reaſon to make a doubt of that order. So that nowe for Numeration, I am clearely ſatiſfied: ſo that onely practiſe ſhall make me fully ready and expert in it. And in the meane ſeaſon, I deſire to learne the other kindes of Arithmetike.

Mayſter. That is well ſayde: but what ſhould you next learne, can you tell?

Scholer. I remember, you ſayd that Addition was next.

Mayſter. Euen ſo, and what that is, muſt you firſt knowe.

ADDITION.

Addition is the gathering together and bringyng of twoo numbers or more, into one totall ſumme: as if I haue 160 Bookes in the Latyne tongue, and 136 in the Greeke tongue, and woulde know how many they bee in all, I muſt write thoſe two numbers, one ouer an other, wryting the greateſt number

highest, so that the first figure of the one, be vnder the first figure of the other. And the seconde vnder the seconde, and so forth in order.

When you haue so done, draw vnder them a right line, then will they stande thus.

1	6	0
1	3	6

Now beginne at the first places, toward the right hand alwaies, & put togither the .ij. first figures of those two numbers, and looke what cõmeth of them, write vnder them, right vnder the line. As in saying, 6 and 0, is 6. Write 6 vnder 6: as thus.

1	6	0
1	3	6
		6

And then go to the seconde figures, and doe likewise: as in saying, 3 and 6 is 9: write 9 vnder 6 & 3, as here you see.

1	6	0
1	3	6
	9	6

And likewise doe you with the figures that be in the third place, saying: 1 and 1 bee 2: write 2 vnder them, and then will your whole sum appeare thus.

1	6	0
1	3	6
2	9	6

So that now you see, that 160, and 136 doe make in all, 296.

Scholer. What? this is very easy to doe, me thinketh I can doe it euen sithe.

There came through Cheapeside two droues of cattell: in the first was 848 sheepe, and

and in the seconde was 186 other beastes.

Those two summes I muste write as you taught mee, thus.

848
186

Then if I put the two first figures togither, saying: 6 and 8 they make 14. That must I write vnder 6 and 8, thus.

848
186
14

Mayster. Not so, and here are you twyse deceyued. Firste, in going about to adde togither two summes of sundrye things, which you ought not to doe, except you seeke onelye the number of them, & care not for the things. For the summe that shoulde resulte of that addition, shoulde be a summe nother of sheepe nor other beastes, but a confused summe of bothe. Howbeit sometimes yee shall haue summes of diuers denominations to bee added, of which I will tell you anone: but firste I will shewe you, where you were deceyued in an other poynte, and that was in writing 14, (which came of 6 and 8) vnder 6 and 8, which is vnpossible. For howe can two figures of two places bee written vnder one figure and one place?

Scholer. Truthe it is: but yet I did so vnderstande you.

Mayster. I sayd in deede that you shoulde write that vnder them, that did resulte of them bothe togither: whiche saying is alwaies true, if that summe bee not exceede a Digit. But if it bee a mixt number, then must you write the diget of it vnder your figures, as I haue saide before: but and if it be an Article, then write o vnder them, and in bothe sortes you shall keepe the article in your minde. And therfore when you haue added your secōd figures, whiche occupy the place of tennes, you shall put that 1 thereto, whiche you kepte in your minde: for though it were ten in deede, yet in that place it is but as one, bicause that euery 1 of that place is ten, for it is the place of tennes. And in like manner: if you haue in the seconde place so greate a number, that it amounteth aboue 9, then write the digites, and reserue the article in your minde, euer adding it to the next place following: and so of all other places, howe manye soeuer you haue. And if you haue a mixt number, when you haue added your laste figure, then write the digite vnder the laste figures, and the article in the nexte place beyonde them: so shall your number resulting of addition, haue one place more than the numbers whiche you should adde togither.

 Scholer.

Scholer. Now doe I perceyue you, and the reason of this is, (as I vnderstande) bycause that no one place can containe aboue 9, which is the greatest figure that is, & then all tennes or articles muste bee put to the next place folowing: for euery place (as I may see) exceedeth the other place next before him, by x.

Nowe (if it please you) I will returne to my example of cattell. But I remember you sayde, I myght not adde summes of sundry things togither, and that myght I see by reason.

Mayster. Truthe it is, if you seeke the due summe of any thing, but if you onely seeke a bare summe, & haue no respect to the thing, then were it better to name the summe onelye without any thing, as in saying 848, without naming sheepe or any thing els. And likewaies 186, naming nothing.

Now let mee see: how can you adde those two summes.

Scholer. I must first set them so, that the two first figures stande one ouer an other, and the other eche one ouer his fellowe of the same place: then shall I drawe a lyne vnder them bothe. And so likewise of other figures, setting alwaies the greatest number highest.

thus

thus,

848
186
―――

Then must I adde 6 to 8. whiche maketh 14, that is mixt number: therfore must I take the diget which is 4, & write it vnder 6 & 8 keping ÿ article 1 in my mind thus.

848
186
―――
4

Next that doe I come to the second figures, adding them togither, saying: 8 and 4, make 12, to whiche I put the one reserued in my mind, and that maketh 13. of whiche number I write the diget 3 vnder 8 and 4, and keepe the article in my minde thus.

848
186
―――
34

Then come I to the thirde figures saying: 1 and 8, make 9, and 1 in my minde maketh 10. Sir shall I write the cipher vnder 1 and 8? Ma. Yea.

Scholer. Then of 10 I write the ciphar vnder 1 and 8, and kepe the article in my mind.

Mayster. What needeth that, seeing there followeth no more figures?

Scholer. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the laste figures, I muste write the ciphar vnder them, and the article in a farther place after the ciphar, thus.

848
186
―――
1034

Mayster. So nowe ye see, that of 848, and 168 added togither, there amounteth 1034.

Scholer. Nowe I thinke I am perfite in Addition.

Ma. That will I proue by this example.

There are two armies of souldiours: in the one are 106800, and in the other 9400: How many are there in both armies say you?

Scholer. First I set them one ouer an other, beginning with the firste numbers at ye right hand, thus. But the nether number will not match the ouer number.

106800
9400

Mayster. That forceth not.

Scholer. Then do I adde 0 to 0, and there amounteth 0, that muste I write vnder the firste place, thus.

106800
9400
0

Mayster. Well sayde.

Scholer. Then likewayes in the seconde place I adde 0 to 0, and there riseth 0, which I write vnder the seconde place thus.

106800
9400
00

Then I come to the thirde place saying: 4 and 8 make 12, of which I write the digette 2, and keepe the article 1 in my minde, thus.

106800
9400
200

Then

 57

Then adde I 9 to 6, whiche maketh 15, to that I adde the article, that was in my minde, and it is 16. I write 6 vnder 6 and 9, and keepe one in my minde, thus.

```
106800
  9400
------
  6200
```

Mayster. Why do you not write both figures seeing you are come to the last couple of numbers?

Scholer. Nay, reason sheweth mee that I must adde that article that is in my minde, vnto the next figure of the ouer summe, thoughe there be no more in neyther summe.

Ma. That is well considered: then doe so.

Scholer. Then say I, 0 in the ouer summe, and 1 in my minde, maketh 1, that I write vnder 0: Then followeth there yet 1 more in the ouer summe whiche hath none to be added to it, for there is none in the nether summe, nor yet in my minde, therefore I thinke I muste write that euen as it is.

Mayster. Yea.

Scholer. Then doth my whole summe appeare thus.

Mayster. If you marke this, you haue learned perfectlye the common addition of all summes

```
106800
  9400
------
116200
```

which are of one denomination: so that ye obserue this also, that in Addition you must haue two numbers at the least, or else howe can you say that you do adde? And euer let the greatest number bee written highest, for that is the best way, though it be not necessarie.

And forget not this, that if you haue many numbers to adde togither, you shal haue oftentymes an Article of a greater value than 10: somtimes 20, sometimes 30, sometimes more, yea peraduenture 100. Therfore as you did wh the article 10, so do with them, reseruing them in your minde, and adding to the number next folloWing so many as their valuer or value certaine is: that is to say, 2 for 20, 3 for 30, and so forth of other. But if the article be 100, then must you not adde the article to the next figures folowing, but to the third figures from them, as I will shewe you anon by example. And if it chaunce the number to bee suche that it doe comprehende two sundrie articles (that is one of tennes, and another of hundredes) then muste you reserue them bothe in youre minde, and adde the article of tennes to the figures that followe nexte, and the article of hundreds, to the figure of the thirde place from thence.

Nowe

Nowe take this example for all. I would ad these xiij. summes in one, whiche I sette after this manner. Then do I beginne and gather the summe of the firste figures, whiche commeth to 107. For first I take 9 there x. times, and that is 90: then 9 and 8 is 17, that is in all 107, of whiche summe I write the 7 vnder the firste figures, and then haue I an article of an hundred in my minde, whiche eyther I muste keepe in my minde tyll I come to the thirde figures, whiche are in the roomes of hundredes, or else I maye for feare of forgetting, write this one (being of the thirde place in your ofcome, vnder the thirde rowe of figures, making two lines, as you see here done. And then must I write the digites vnder the lowest lyne: and this is the surest way, when the summe is so greate, that the addition of one rowe passeth 100.

4889
4599
2299
3699
2399
4090
1099
3198
299
699
899
499
589
[illegible]

When I haue so done, I muste then come to the seconde rowe of figures, and adde them togither, whiche dothe make 115, of whiche

ſumme I write the dygitte 9 vnder the ſame ſecond rowe, and then I haue a mixt number remaining of two figures, of which the 1 (that ſtandeth for 10) muſte bee added to the ſecōd or next place after them that I did laſte adde. And the other (that ſtandeth for 100) muſt be added to ẏ third place from thence.

Scholer. That is to ſay, the fourth place from the firſt line or rowe of figures.

Maiſter. Euen ſo. And thē will the ſumme appeare thus:

4	8	8	9
4	5	9	9
2	2	9	9
3	6	9	9
2	3	9	9
4	0	9	0
1	0	9	9
3	1	9	8
	2	9	9
	6	9	9
	8	9	9
	4	9	9
	3	8	9
1	1		
	1		
		5	7

Then adde the thirde rowe of figures, with the two vnities betweene the lyne, and the ſumme amounteth to 50: of which I write the Cypher vnder the ſame thirde rowe, and the 5 vnder the nexte figures toward the lefte hande. And with my pen I giue a daſhe to the two vnities betweene the lynes, whoſe valew I haue already added vnder the loweſt lyne.

Then I adde the figures of the fourth rowe, with the 1 and 5 that are vnder them betweene

the

the two lynes, and they make 29: then dashe I the 1, and the 5, with my penne, as I did before ye two vnits: and so write vnder the lowest line the 9 (that is the digit) vnder the fourth place: and the 2, that is the article, beyonde it, towarde the lefte hande. So those summes doe make 29057.

4889
4599
2299
3699
2399
4090
1699
3118
299
699
899
499
389
11
51
29057

Scholer. This seemeth somewhat harde, by the reason of so many numbers togither. Howebeit I thinke if I doe often proue euen with this same example, I shall be able to doe so, shortly, wt any other summe.

Mayster. So shall you. For it is often practise that maketh a man quycke and ripe in all things: But bicause of such greate summes there may chaunce to bee some errour, I will teache you how you shall proue whether you haue done well or no.

Scholer. That were a greate helpe & ease.

Mayster. Beginne first wt the highest number, and then to all the other orderly, and adde them togither, not hauing regarde to their places, but as though they were all vnities:

and still as youre number encreaseth aboue 9, cast away 9. Then go forthe, euer casting a⸗way 9, as often as it amounteth thereto: and so doe till you haue gone ouer all the numbers that you intended firste to adde, and whatsoe⸗uer remaineth after suche addition and casting away of 9, write it in some voyde place by the ende of a line for the better remembraunce: and then put togither the figures that resulte of the addition, still casting away 9 also. And then that that remaineth, write at the other ende of that line: and if those two figures be like, then haue you well done by likelyhode: but if they bee vnlike, then haue you missed. As for ex⸗ample in this present summe: The firste fi⸗gure of the ouer lyne is 9, let him go: then 8 and 8 is 16, take away 9, and there remayneth 7, adde to it 4 that followeth, and that ma⸗keth 11, from which if you take 9, there resteth 2: then come to the next rowe, whose first and second numbers are 9, therefore ouerpasse them bothe, and take ẏ 5 to the 2 which did remaine in the first rowe, that maketh 7, put thereto the 4 following, that maketh 11, thence take 9, and there remaineth 2: next that, go to the thirde line, whose two firste numbers you maye let passe, bicause they are nynes: then take the two

two whiche wt the other two ỹ remained in the second row, make 6: then go to the fourth row, whose two first numbers let go, and take the 6 to the 6 that remained, and that maketh 12, take away 9, and there resteth 3, which with the 3 that is next maketh 6. And so go through all the other numbers, and you shall finde that there remaineth 5, after you haue cast awaye 9 as often as you finde it: therefore write 5 at one ende of a line in a voyde place, thus. 5——
Then gather all the figures of the totall summe which is vnder the lowest line, and cast away 9 as often as you finde it, as thus: seuen and 5 make 12, take away 9, and there resteth 3, to that if you adde the 2 that is last (for you may let go the 9) then doth it make 5, which you must wryte at the other ende of the line that you made in the voyde place, and it will bee thus. 5———5

And then you see ỹ those two figures bee like, whereby you may knowe that you haue done well, and so may you proue in any other.

Scholer. If it please you, I will proue in an other summe.

Mayster. With a good will.

Scholer. Then will I take one of youre former examples, which was this.

First

First in the highest line, 8 and 6 make 14, then 9 taken awaye there remaine 3, to whiche I adde the 1 that followeth, and that maketh 6. Then come I to the second line, where I finde firste 4, whiche with 6 maketh 10, from that I take 9, and there resteth 1, the next figure is 9, and therefore I let him alone, so finde I one remaining, which I set at the ende of a line thus, 1———

	106800
	9400
	116200

Then I come to the totall summe, and there I finde that all the figures put togither, make 10, from which I take 9, and there resteth 1 also, whiche I put at the other ende of the line thus, 1———1

And bicause they bee like, I know that I haue well added.

Mayster. So you knowe nowe both howe to adde two summes or more togither: and also howe to proue whether you haue done well or no: whiche thing also you may doe best by Subtraction. But bicause you cannot yet skill of it, I will let that passe till anon, and will teach you now how to adde summes of diuers denominations: whiche thing can neuer bee, but when the one denomination is such that it conteyneth the other certayne tymes. And yet you

Addition of numbers of diuerse denominations.

you shall adde them to the other, not after this sort as you did them that were of one denomination, but after such a sort as I will nowe shew you, that is to say.

If you haue a summe of diuerse denominations, then loke that ye set euery denomination by him selfe, with some note or figure of his denomination, as they be wont to bee written. Then write your other summes so vnder that first, that euery one bee set vnder the other of the same denominations, as for example: if your denominations be poundes, shillings, and pens, write poundes vnder poundes, shillings vnder shillings, and pens vnder pens, and not shillings vnder pens, and pens vnder poundes.

Scholer. Nowe that you haue spoken it, me thinketh it needeth not to warne mee of it, for it were against reason to cõfound summes: so but yet if you had not spoken of it, peraduenture I should haue beene deceyued in it.

Mayster. If you doe say it is so plaine, I will speake no more of it, but with an example make the matter to appeare euidently.

First, one man oweth mee 22 li, 6 s, 8 d. An other oweth mee 5 li, 16 s, 6 d. And an other oweth mee 4 li, 3 s, I woulde

know what this is altoge-
ther. Therefore must I first
set down my greatest sūme
and thē the other, euery one
vnder his denomination a-
greeing to the greatest summe, as here you see.

li	ß	d
22	6	8
5	16	6
4	3	

Then must I begin at the smallest nūbers, (which must alwayes be set next ỹ right hand) and adde them togither, and if the summe of them will make one of the next denominatiō, then must I keepe it in my minde till I come to that place, or else for more easinesse, write it vnder that place betweene the double line, and vnder that first place must I note the residue, if there remayne any of the same denomination, but if there remaine none, then neede I to write vnder it nothing. And this is all that you must marke in this Addition: for all other things are like to the other maner of addition before mentioned. Therefore the chiefest point of this addition is, to knowe the values of common coynes and rated summes. As howe many shillings be in a pound: how many pens in a shilling, of which and of other like things I will instruct you hereafter, in teaching of Reduction: But nowe I maye not disturbe youre witte from the thing that wee are about

aboute.

Therefore let vs returne to that former example, which I proposed of three detters, which summes when I had sette orderly, they stode thus with a double lyne vnder them.

℔	ß	d
22	6	8
5	16	6
4	3	

Then to adde thẽ vnto one summe, I muste beginne at ye right hand, where the smallest denomination is, and adde them togither firste, saying: 6 and 8 make 14. Nowe seeing these 14 are pennies, and that 12 pens make one shilling, which is the next valewer, I take away 12 from 14, & there resteth 2, which I write vnder the pennies, and for the other 12, which maketh 1 shilling I write 1 vnder the tytle of shillings, thus:

℔	ß	d
22	6	8
5	16	6
4	3	
	1	
		2

Then doe I adde all the shillings togither, & finde them 25, to which I adde ye 1, betweene ye two lynes, yt maketh 26, but bycause that 20 shillings do make 1 pounde, I take away 20

℔	ß	d
22	6	8
5	16	6
4	3	
1	1	
	6	2

from

from 26, and for that 20 I write 1 vnder the poundes betweene the two lines, and the other 6 that remayneth, I write vnder the shillings, as appeareth in the example before.

Then come I to the poundes, adding them all togither, and finde them to bee 31: thereto I adde the 1 betweene the two lynes, & that maketh 32. which summe I write downe whole, bicause there resteth no greater denomination, and then my whole summe appeareth thus.

li	ß	d
22	6	8
5	16	6
4	3	
1	1	
32	6	2

So is my totall summe 32 li. 6 ß. 2 d. And this may you proue in an other like summe.

Scholer. Then will I caste the whole charge of one monethes commons at Oxforde with batteling also.

Mayster. Go to, let mee see howe you can doe?

Scholer. One weekes commons was 11 d.ob.q̃.and my batling that weeke was 2 d. q̃.q. The second weekes commons was 12 d, and my batling 3 d. The thirde weekes commons 10 d.ob. and my batling 2 d.q.c. The fourth weekes commons 11 d, q̃, and my batling

ling. d. ob. c. These s summes wold I adde into one whole sūme, and therfore I will set them one ouer an other, thus.

d.	ob.	q̃.	q.	c̃.
11	1	1		
2		1	1	
12				
3				
10	1			
2			1	1
11		1		
1	1			1

But I had forgotten, I shoulde haue set ẏ greatest sum highest.

Mayster. So is it commonly beste, howbee it, here it forceth not: and in suche summes as this is, that go by order of weekes, dayes, or yeares, it is better to keepe that order, than to alter them, and to set ẏ greatest number highest, for that serueth for suche summes as go not by order.

d.	ob.	q̃.	q.	c̃.
11	1	1		
2		1	1	
12				
3				
10	1			
2			1	1
11		1		1
1	1			
			1	

Scholer. Then yf I haue sette them well ynough, I will beginne to adde them thus.

Firste of the smallest balewers at the right hande, whiche are called cees, I finde 2, & seeing ẏ 2 cees, we make one

q, I will write nothing vnder the cees, but will write 1 q for 2 cees, vnder the kewes betweene ye lines, as the exãple on ye other side sheweth.

Then come I to the nexte valewers, where I finde 2 q, and to them I adde the q that is betweene the lynes, and so are they 3 q: but bycause 2 q, maketh one q̄, I write one q̄ vnder the farthings betweene the lines, and the q that remaineth, must I write benethe the nethermoste line vnder the kewes, thus.

d.	ob.	q̄.	q.	c
11	1	1		
2		1	1	
12				
5				
10	1			
2			1	1
11		1		
1	1			1
		1	1	
			1	

Then come I to the farthings, where I finde 3, and the other q̄ that is betweene the lines, maketh 4 farthings. And bycause 4 q̄, make iuste 1 peny, I shall write nothing vnder the farthings, but muste write 1 vnder the pens, betweene the lynes.

Next that must I adde the halfe pens togither, of which there are 3. but seeing that 2 ob. make 1 d, I muste write 1 vnder the pens betweene the lynes: but howe shall I doe it, for

there

there is 1 alreadie?

Mayster. Haue you forgotten how I did in addition of the greate summe before? you must set it vnder the other, so shall they bothe stande for 2. For if you shoulde set it before or behinde the other, they should make 11.

Scholer. I remember it nowe, and I perceyue the reason. Then I will write 1 ob. vnder the halfepens, and for the other two halfepens, whiche make 1 d., I write 1 vnder the pens: Then come I to the pens, and finde that there are of them 52, then put I to them the 2 betweene the lines, and that maketh 54, which amounteth to 4 ß. 6 d.: the 6 d. I must write vnder ẏ pens, and the 4 ß. I must sette (I suppose) farther toward the left hand by them selfe.

Ma. Euen so.

Scholer. Then appereth all my addition, thus. And ẏ summe is 4 ß 6 d. ob q.

ß	d.	ob.	q̄.	q.	ƺ.
	11	1	1		
	2		1	1	
	12				
	3				
	10	1			
	2			1	1
	11		1		
	1	1			1
	1		1	1	
	1				
4	6	ob.		q.	

Ma. Now haue

 you

you done this well. But tell me, why did you write xxwe, cce, thus q, c, and not rather thus q, as the fashion is ?

Scholer. Bicause I thought it was the best way for due gathering of euerie denomination by him selfe.

Mayster. So was it in deed. Well now, can you tell howe to proue this addition, and such other like of diuerse denominations', and to trie whether you haue done well or no ?

Scholer. I would I could.

Mayster. That shall you doe by this meanes. First as you did begin to adde, so recken againe euery denomination by it self, and when you finde so many small that we make any other denomination, let them goe, and keepe in mind only the residue that will make no greater denomination, and looke whether there bee any such like value vnder the nether line, and if there bee, you haue well done, and so goe forth from one denomination to an other, vnto the ende.

But heere must you note, that in gathering of the summes, yee muste recken those figures that are written betweene the lynes, with them that are written aboue them: as for an example. I will examine that summe that I did

did last adde, which stode thus.

li	s	d
22	6	8
5	16	6
4	5	
1	1	
32	6	2

Firste I finde 6 and 8, whiche maketh 14, from whiche I take 12, bycause it maketh one of the next denominations, and there remayneth 2, and vnder that place I see a like figure, therefore I knowe that well to be done. Then come I to the s, where I finde 1, 3, 16, and 6, that maketh 26, I cast away 20, for they make an other demomination, that is to say poundes: and the 6 whiche remayneth, is like to the 6 that is written vnder them beneath the lowest line, therefore that is well done also. And thence I go to poundes, where I finde 1, 4, 5, 22, that is 32, to whiche summe agreeth an other like vnder it. Therefore I iudge all well done.

Scholer. I perceyue reason in this probation. Nowe will I attempt the same in the summe that I did adde, whiche when I had ended adding, stoode as you maye see in the page following.

Firste amongest the cees I finde but two, whiche make one q euen, therefore there muste nothing bee vnder the lyne for them: And

ß	d.	ob.	q.	q.	f.
	11	1	1		
	2		1	1	
	12				
	3				
	16	1			
	2			1	1
	11		1		
	1	1			1
	3		1	1	
	1				
4	6	ob.		q.	

amongst the kewes I finde 3, of which two make ob, therfore I let them go, and the one q, that is lefte, hath an other lyke vnder his place, therefore that is well done.

Then the farthinges are iust 4, whiche make 1 d, & therefore I let them go: Amongst the halfe pens there is one odde (for 2 must I cast awaye, bicause they make one penny) and vnto it answereth a like sum vnder it. The pens are 54, from whiche I take away 48, that makes 4 ß, and the 6 remayning agree to a like figure set vnder them. And last of all remayneth the 4 ß, whiche the abiected pens did make: so I perceyue that I haue well done. Nowe this will I not forget. But will this examination serue in all Addition?

☞ Mayster. It serueth for all addition of sundrie denominations, if the Addition be made with two lines, (as were these) else it will not

serue,

ferue, bicause that those summes which are heere added betweene the lynes, in Addition by one lyne, are understanded and not written: but I let ÿ waies passe, bicause as it is cõmon so is it more deceyuable thã this waies, namely if a mans memory be either dul or troubled.

Scholer. Yet it were good to knowe that waies also.

Mayster. If you desire to know it, this it is in fewe wordes. Doe euery thing as you did in this sorte of Addition, saue that where you made here two lynes, you shall make there but one: and those summes that you did here write betweene the lines, you must keepe in your memory, and vse them (as you did here) eche one when you come to his place. An other forme of Addition.

Scholer. Then they differ not, but in this, ÿ this addition wt two lynes leaueth nothing to memory, but writeth downe all: and the other way cõmitteth certaine numbers to memory, as you taught me in the firste examples of addition of small summes of one denomination. But what if a man vse it (as you say men do commonly) how shall it be examined?

Maister. Seeing you are so desirous of it, I will shew bothe an example of the addition and also the manner to examine it.

I propose these three summes to bee added, and I gather firste the pence, as I did in the other sorte, and I finde of them 8, 3, 9, that is 20, of which summe I bate away 12, which make 1 ß, and keepe that 1 in my minde, and the rest, that is 8, I write vnder the pence.

℔	ß	d
12	8	9
6	7	3
3	6	8

Then doe I adde the shillings togither, and finde of them 6, 7, 8, that is 21, whereof I bate 20, that make 1 ℔, which I keepe in minde, and to the other 1 that remaineth, I adde that one that came of the pens, & was in my minde, which make 2, and them I write vnder the shillings.

Then doe I recken the poundes togither, 3, 6, 12, that is 21, and to them I adde the 1 in my minde that remayneth of the shillings, which make 22, them doe I write vnder the poundes, and then my summe totall appeareth to bee 22 ℔, 2 ß, 8 d.

℔	ß	d
12	8	9
6	7	3
3	6	8
22	2	8

n other
rme of
oofe.

Now to examine this summe and all such like, you shall doe thus. Firste beginne at ẏ left hand with the poundes, and take from them that are aboue the lyne, 9, as often

as

as you can: then that that remaineth, shall you double, and ioyne it with the shillings, & take away 9 from that as often as you can, and whatsoeuer remaineth, ye shall take for it three times so much, and put to the pence: then take from all that sume 9, as often as you can, and what so remaineth after you haue withdrawē 9 as often as you can, write that at the ende of a line, as I taught you in the other addition.

And then come to the summe vnder the line, beginning with the poundes, and doe euen as you did with the summes aboue the lyne, tyll you come to your pennies: and if the figure of the summe that remaineth after casting away 9, (as often as you can) doe agree with the other that remained before of the other summe, which you did write at the end of the line, then haue you done well, els not: and for an example, I will examine that laste summe which was thus.

First I shall begin at the left hand with the poundes, putting thē togither, which make 21, in which summe I finde 9 twyse, (for twyse 9 is 18) that I deduct, and there remayneth 3: that 3 muste I double (as

li	s	d
12	8	9
6	7	3
3	6	8
22	2	8

I saide) bicause it is the remainer of ẏ poũdes, and it will bee 6. Then gather I the summe of the shilling, which is 21, to the which I adde the aforesaide 6, and then is it 27, wherein I finde 9 three tymes, and there remayneth nothing. This remayner shoulde I take three tymes, but three tymes nothing, is nothing: therefore in this place is there nothing left to bee added to the pennyes. Wherefore I must take the summe of pennies alone, which is 20, from thence if I take 9 twice, there remayneth but 2, which I put at the ende of a lyne, thus. 2————

Then I come to the poundes of the vnder number or totall summe, and there I finde 22, from which I take away 9 twise, and there remayneth 4: that 4 I double, and it is 8, then doe I adde that 8 to the shillings, and it maketh 10, from which I withdrawe 9, and there resteth one: then doe I take that 1 three tymes, and it maketh 3, which I adde to the 8 d, and it maketh 11, from which if I bate 9, there resteth 2, which is equall to the number noted at the ende of the line: & thereby I perceyue that I haue done well.

Scholer. But I doe not see the reason of this.

Mayster.

Mayster. No? no more doe you of many things else, but hereafter will I shewe you the reasons of all Arithmeticall operations. For this I iudge to bee the best trade of teaching, first by some briefe preceptes to instruct a learner somewhat in the use of the arte, before hee learne the reasons of the arte, & then may you afterwarde more sooner make him to perceyue the reasons: for hard it is for to occupie a yong learned witte with both the arte and the reasons of it all at once: howbeit hee shall neuer bee cunning in deede in an arte, that knoweth not the reason of euerie thing touching it. But for this worke, bicause the reason is easie, I will shew if you likewe. You know that if one pound do remaine, it beeing turned into shillings, woulde make 20 ß, in whiche number there is 9 conteyned twise, and 2 ß beside. And therefore for one pounde you shall take 2 ß, and so for euerie one pounde 2 ß.

The reason of this proofe.

The best trade of teaching.

Scholer. I see it well, for if there remayned 7 ℔, after the nynes were cast away, I must take 14 ß, for that 7 ℔. And so haue I caste away 14 times 9 ß, and yet remayneth of euery pounde 2 ß, which maketh 14 ß.

Mayster. Likewayes in shillings, which containe 12 d: for euery shilling, if you abate

9 pens, there resteth 3 pens.

Scholer. It is plaine ynough. And so if 5 shillings do remaine, I muste take it for 15 d, that is 3 pens for euery shilling, and yet in that so doing, I haue caste away fiue tymes nine pens.

Mayster. Other workes haue as good reason, but I will not stande about yelding reasons nowe.

Scholer. Yet one thing more I pray you shew me, why did you write your number that remained (after you had withdrawne all the nynes) at the ende of a line? for I saw no reason why that line did serue.

Mayster. Did you euer marke a Carpenter when he wrought?

Scholer. Yea many tymes.

Mayster. And haue you not sene him when he hath taken measure of a boorde, that he hath pricked it, and hath with a twitch of his hande drawne a line from the pricke that he made?

Scholer. Yes I haue marked that, & haue seene some make 3 or 4 lines, by the pricke, and some also haue I seene make a crosse by it, but that I perceyued was for the easie finding of their pricke.

Mayster. And euen so is this line for the

easie

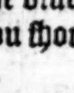
easie finding of your remayner, and therefore some doe make a crosse, thus.

And set the one remainer aboue the crosse, & the other vnder the nether part of the crosse, as if I shoulde set my two remainers thus.

But there is an other sort of proofe of Addition, to which the crosse serueth more meeter: and that is whē the addition is of diuers denominatiōs, and I would examine euerie denomination by it self, which wayes though it be not much vnlike to the first proofe that I brought of suche diuerse summes, yet I will declare it, least you should thinke that I would hide it from you. An other sort of proofe.

You must make so many lines in your crosse, as you haue sundrie denominations: as if you haue but two denominations, then you maye make it thus, that the ouer part and the nether part may serue for one denomination, and the two sides for the other.

And if you haue three denominations, as poundes, shillings, and pennies, then must you make three lines thus. The vpright line may serue for poundes, and the highest thwart line for shillings, and the lowest for pens: as for



example

example I will take a ſumme thus added.

li	ſ	d
16	12	5
12	8	1
9	2	7
37	3	1

2
3 + 3
1 + 1
2

For the proofe of the which, bicauſe it conteyneth three denominations, I muſt make a croſſe of three lines, thus. Then I recken firſt at the right hand the pennies: 7, 1, 5. make 13, from whiche I take 12 for the next denomination, that is to ſay, a ſhilling, and there reſteth 1, whiche I muſt write at one ende of the nether thwart line.

After that I gather the ſumme of the ſhillings, 2, 8, 12, whiche maketh 22, to them I put one that I tooke of the pennies, and that maketh 23: from thoſe I take 20, the quantitie of the next greater denomination, that is to ſay, a pound, and there reſteth 3, whiche I write at the ende of the higheſt thwart line.

Thirdly, I adde togither the poundes, 9, 12, 16, whiche make 37, to them I adde the 1 that came of the ſhillings, and then there is 38, wherein I finde 4 times 9, and 2 ouer, that 2 I write on the vpright line.

That done, I come to the totall ſumme, and

and examine it, beginning at the pennyes, where I finde but one, and can not take 9 from him, therefore I set him at the other ende of the nether thwart line: Then come I to the shillings, where I finde onely 3, which bycause it is lesse than 9, I sette it at the other ende of the line of shillings, that is, the outermost thwart line.

Laste of all, of the 38 li, I take out foure times 9, which is 36, and there remaineth 2, which I write vnder the vpright lyne.

Then I consider euery number, comparing it to the number that is against it, and bicause I finde thē to bee euery one like his match, I know that I haue well done.

Scholer. This crosse I perceyue doth serue for those three denominations, poundes, shillings, pennyes, but what if it had, ob, q̄, q, and c?

Mayster. You thinke you bee at Oxforde still, you bring forth so faste your q, &c. These lines, as I haue sayde, do serue for three denominations, such as they bee: as here they doe serue for poundes, shillings, and pennies: but if ye haue no poūdes in your summe, then may they serue for shillings, pennies, and halfepennies, yea for q̄, q, and c, if you haue no greater

ter denomination, so that you remember that the upright lyne serueth for the greatest denomination, and the highest thwart line, for the next, and the lowest for the least.

And so if you haue foure denominations, you must make your crosse with so many lines. And if that your summe bee of more denominations, make so many lines in youre crosse. And thus will I make an ende of Addition.

Examples of Addition.

lb.	s.
262587	6
41635	12
28124	2
471	4
332818	4

lb.	s.	d.
340	17	10
28	6	8
13	13	4
382	17	10

The proofes.

An other example.

lb.	s.	d.	q.
22		6	2
2	3	4	
	10	2	3
24	14	1	1

Sub-

SVBTRACTION.

Scholer.

Then haue I learned the two firste kindes of Arithmetike: now as I remember, dothe followe Subtraction, whose name mee thinketh wthe sounde contrary to Addition.

Mayster. So is it in deede: for as Addition encreaseth one grosse summe by bringing many into one, so contrary waies, Subtraction diminisheth a grosse summe by withdrawing of other from it, so that Subtraction or Rebating is nothing els, but an arte to withdrawe and abate one sume from an other, that the Remainer may appeare. Subtraction.

Scholer. What call you the Remayner?

Maister. You may perceyue by the name.

Schol. So me thinketh: but yet it is good to aske the trouth of all such things, least in trusting to myne owne coniecture, I bee deceyued.

Mayster. So is it the surest way. And as

I see cause, I will still declare things vnto you so plainly, ỳ you shall not neede to doubte. Howbeit, if I doe ouerpasse it sometimes (as the mãner of men is to forget the small knowledge of them to whome they speake) then doe you put mee in remembraunce your selfe, and that way is surest.

Remainer. And as for this worde that you laste asked mee, take you this description. The Remayner is a sum left after a due Subtraction made, which declareth the excesse or difference of the two other numbers: as if I woulde abate or subtracte 14 out of 18, there shoulde remayne 4, which is called the Remayner, and is the difference betwene those two numbers 14 & 18.

Scholer. I perceyue then what Subtraction is: Nowe resteth to know the order to worke it.

Maister. That shall you doe by thys meanes. Firste you must consider, that if you should go about to rebate, you must haue two sundry summes proposed, the firste which is your grosse summe or summe totall: (and it must bee set highest) and then the rebatement or summe to be withdrawen, which muste bee set vnder the first (whether it bee in one parcell or in many) and that in such sort, that the first

figures

figures be one iust ouer an other, and so the seconde and thirde, and all other following, as you did in Addition: then shall you drawe vnder them a line, and so are your summes duly set to begin your working.

Then begin you at the right hande (as you did in Addition) and withdrawe the nether number out of the higher, and if there remaine any thing, write that right vnder them beneath the line: and if there remaine nothing (by reason that the 2 figures were equall) then write vnder them a cypher of nought. And so do you with all the other figures, euermore abating the lower out of the higher, and write vnder them the Remayner still, till you come to the ende. And so will there appeare vnder the line what remayneth of your grosse summe, after you haue deducted the other summe from it, as in this example.

I receyued of your father 48 ß, of whiche I haue layde out for you 36 ß: now woulde I knowe what doth remaine? and therefore I set my numbers thus in order: First I wryte the greatest summe, and vnder him the lesser, so that the figures at the right side bee euen one vnder another, and so the other, thus.

ß
48
36

Then doe I rebate 6 out of 8, and there resteth two, whiche I write vnder them right beneath the line, thus.

8
48
36
2

Then I go to the second figures, and doe rebate 3, out of 4, where there remayneth 1, which I write vnder them right, & then the whole summe and operation appeareth thus.

8
48
36
12

Whereby it appeareth, that if I withdraw 36, out of 48, then remaineth 12.

Scholer. Nowe will I proue in a greater summe: And I will Subtract 2367924 out of 3468946. Those summes I set in order thus.

3468946
2367924

Then doe I begin at the right syde, and deducte 4 out of 6, and there resteth 2, whiche I write vnder them. Then goe I to the second figures, and withdraw 2 out of 4, and there remaine two, which I set vnder them also: then I take 9 out of 9, and there resteth 0, whiche I write vnder them, for you say, that if the figures be equall so that nothing remayne, I must write this ciphar 0 vnder them.

Mayster. It was well remembred, nowe

go forth.

Scholer. Then come I to the fourth place and draw 7 out of 8, and there remayneth 1, whiche I write vnder them also. Then in the fift place I take 6 from 6, and there resteth nought, for it I write vnder them a cipher, 0: Then in the sixt place 3 rebated from 4, there remayneth 1, whiche I write vnder them: and likewise in the vij. and last place, 2 taken from 3, there is left 1: whiche I write vnder them, so haue I done my whole working, & my summes appeare thus.

3468946
2367924
1101022

Whereby I see, that if I rebate 2367924, out of 3468946, there remayneth 1101022.

Mayster. This is well done. And that you may bee sure to perceyue fully the arte of Subtraction, let me see howe can you subtract 52984732 out of 8250003456.

Scholer. Firste I set downe the greatest summe, and after that I write vnder it the lesser number, beginning at the right side: and then my figures will stande thus.

8250003456
52984732

Then take I 2 from 6, and the reste is 4,

whiche I write vnder them: then do I withdrawe 3 from 5, and there remaine 2, which I write vnder them. Then take I 7 out of 4, but that I can not, what shall I now do?

Note. Mayster. Marke well what I shall tell you now, howe you shall doe in this case, and in all other like. If any figure of the nether summe be greater than the figure of the summe that is ouer him, so that it can not be taken out of the figure ouer him, then muste you put 10 to the ouer figure, and then consider how muche it is, and out of that whole summe withdrawe the nether figure, and write the rest vnder them. Can you remember this?

Scholer. Yes, that I trust I shall. Now then in mine example where I shoulde haue taken 7 out of 4 and coulde not, I put 10 to that 4, whiche maketh 14, from it I take away 7, and there resteth 7 also, whiche I write vnder them.

Note. Mayster. So haue you done well, but now must you marke another thing also: that whensoeuer you do put away 10 to any figure of the ouer number, you must adde one still to the figure or place that followeth next in the nether line, as in this example there followeth 4, to which you muste put 1, and make him 5,

and

and then go on as I haue taught you.

Scholer. Then shall I say: 4 and 1 (which I muste put to him for the 10 that I added to 4 before) make 5, which I shoulde take out of 3, but that can not bee, therefore must I put to it also 10, and then it will bee 13, from which I take 5, and there resteth 8 to bee written vnder them: and bycause of that 10 added to the 3, I muste adde 1 to 8 that followeth in the nether lyne, and that maketh 9, which I shoulde take out of 0, and can not, therfore I put therto 10, and that maketh 10: from 10 I take 9, and there remayneth 1, which I write vnder them.

Then doe I adde 1 likewise to the nexte figure benethe, which is 9, and that maketh 10: that 10 shoulde I take out of the figure aboue, but I can not, for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written vnder them. Thē come I to the nexte figure which is 2, and to him doe I adde 1, which maketh 3, that 3 I can not take out of nought, therefore of that nought I make 10, and thence doe I take 3, so remayneth there 7 to bee written vnder them. Likewise doe I put 1 to 5 that followeth, and then is it 6, ẏ woulde I take out of 5, and can not, therefore I adde

10 to that 5, and it maketh 15, from which I rebate 6, and there remaineth 9, which I write vnder them. Now haue I spent all the nether figures, and what shall I doe more?

Mayster. You shoulde haue added 1 to the nexte figure following (if there had been any) bycause you added 10 to the laste figure before of the ouer lyne: but seeing there is no figure following, you muste adde that 1 to the place following, & then deduct that 1 from the number aboue.

Scholer. Then shall I say, bycause I borowed 10 to the ouer 5, I muste put 1 in the nexte place benethe, that is vnder 2: then muste I subtract that 1 from 2, and there resteth 1 to be written vnder that 2 in ye ninth place. Now I haue no more to subtracte, for there is neuer any figure remayning benethe, nother yet any vnitie to bee added, bicause I borowed not 10 to the figure laste before, and yet is there 8 remaining in the ouer lyne, which (I thinke by reason) shoulde bee set at the ende of the figures in the lowest rowe which is vnder the line, for bycause there was nothing taken from it.

Mayster. That is well considered, and reason teacheth so in deede.

Scholer. But syr I beseche you, shall I alwayes

alwayes when any number so remayneth alone (as this 8 did) write him vnder the lyne straight agaynste his owne place?

Ma. Yea, what else? whether they bee one or many: and this well remembred, you haue sufficiently learned Subtraction. How bee it, bicause of certaine things that myght deceyue you, if you did not take good heede to your woorking, I will propose to you an other example of many numbers to bee subtracted, as thus.

I receyued of a friende of myne to keepe 2869 crownes, of which at one tyme I deliuered him agayne 500, at an other tyme 368, and at an other tyme 440, and an other time 80, and an other tyme 64: nowe woulde I knowe howe many doth rest behinde. Therefore first I set downe my grosse summe, and a lyne vnder it: and vndernethe it I set all the parcelles, thus: and vnder them a double line.

```
 2869
 ----
  500
  368
  440
   80
   64
 ----
 1452
```

Then firste I beginne at the first place, and gather togither the summe of all those lynes (saue the ouermoste) in their

 firste

firste figures, and so doe I with all the figures of the seconde place, & so forthe, as I did in Addition, saue that I leaue out the highest rowe of numbers, (as the lyne warneth mee) and that summe so gathered betweene the double lyne, doe I subtract out of the highest rowe of numbers, and the remayner doe I set vnder the nethermoste lyne: as for example.

I set the summes as before: then doe I gather the first figures togither, where I finde but 4 and 8, that make 12, (for three cyphers increaseth no summe in addition, as you learned before) of the 12 therefore doe I write the digit 2 betwene the double lyne, and keepe the article in my minde, till I come to the seconde places, where I finde, 6, 8, 4, 6, that make 24, to them I put the article in my minde, and it is 25, of which I write 5 vnder the seconde place, and keepe the digit 2 in my minde for the thirde place, where I finde 4, 3, 5, that make 12, to the which I adde the 2 in my minde, and that maketh 14, thereof I write the 4 vnder the thirde place: and bycause there

2869
500
368
440
80
64
1452
1417

remayneth no more figures to bee added, I write the digite 1 in the fourth place, as you see in the example.

Then come I to subtracting of this summe betweene the lynes, for by Addition it is equall to the fiue parcels ouer it. Therefore I proceede to subtract it from the ouermost summe, saying: 2 from 9, remaine 7, to bee written vnder them beneath the lowest line. Then in the seconde place I take 5 from 6, and there resteth 1, to be written vnder them. Then in the thirde place, 4 from 8, resteth 4. Last of all in the fourth place, 1 from 2, remayneth 1. And thus I see that after those 5 summes are subtracted from 2869, the Remayner is 1417.

Scholer. This I perceyue: but is there no shorter way and more spedier?

Mayster. Yes, when you are a while exercised in it: for you may as fast as you can gather the numbers togither, withdrawe them out of the highest summe, if so be it, that all the parcelles which you doe gather, doe not exceede nine, but and if they exceede nine, then muste you subtracte onely the digit that is in it, and reserue the article till the nexte place, where you shall adde it with the other figures, and so subtracte the whole out of the figure aboue them

An abridgement of the former maner of Subtraction.

them: but and if in this place the summe of the parcels doe exceede 9, then (as I sayde before) subtract the digit onely, and reserue the article to the next place: and so still go forth, till you haue ended your working.

As for example: in the laste summes proposed, I gather firste in the firste place 4 and 8, that maketh 12, of whiche I deducte the dygit 2 out of 9, and write vnder the remayner, which is 7, & the article 1 I keepe in my mind. Then in the seconde places I gather the parcelles 6,8,4,6, which amount to 24, to that I adde the article 1, whiche I haue in my minde, and then is it 25. Then do I take 5 (that is the digitte in this number) from 6, that is in the seconde place of the highest summe, and there remayneth but 1 to be written vnder them, and nowe do I keepe the article 2 in my mind still. Then in the thirde place 4,3,5, maketh 12. and the article 2 in my mind maketh 14: then take I 4 (whiche is the digit) from 8 that is ouer them, and there resteth 4, whiche I write vnder them. Then haue I the article 1 yet in my minde, which I should adde to the parcels next folowing, but seeing there is no number folowing, I take the digit alone & deduct him out of the next sum aboue, which is 2, & then is the

remayner

remainer 1, whiche I write in the fourth place vnder 2. Lo, now haue you a shorter way.

Scholer. I like both wayes well, and I perceyue both well, yet as in the one the working seemeth somewhat long, so in the other it leaneth very muche (mee seemeth) to remembrance, and therefore may cause errour quickly, except a man haue a quicke and an exercysed remembrance.

Mayster. What? woulde you then haue suche a way as shoulde not bee so long as the one, nor so short as the other?

Scholer. Yea if there were any suche.

Mayster. Then doe thus: still as you gather your parcels, when they exceede a digit, & maketh him 10 or more, take the article, and write him betweene two lynes (as in ẏ firste example) vnder the next place towarde the left hande: and then deducte the digit from the figure that is ouer him, and write the remayner. And then when you gather the next parcelles, you shall adde to them the figure that is vnder them, betwene the two lines. And if it exceede 9, doe as I sayde before, write the article vnder the next place betweene the lines, and subtract the digit from the figure that is ouer those parcels: and if that all the parcels togither and the

the number betweene the lines doe make but a digite, then deducte it wholly from the figure aboue: as in this example. I woulde subtract out of 4030 8964, these three parcelles,

20003428	40308964
10002342	20003428
10101461	10002342
	10101461

Therefore I set them firste in order due: and then I gather the parcelles of the firste place, which are 8, 2, 1, that is 11: of whiche I take away the article, and set him vnder the second place betweene the lynes: and the digit 1 that remayneth, I deduct out of 4, and there resteth 3 to bee written vnder the firste place beneathe the lowest line. Then come I to the seconde place, and gather the parcels of it, 6, 4, 2, & the 1 betweene the lynes, whiche make 13, of whiche I take the article, and set him vnder the thirde place betweene the lines, and the digite 3 I take from 6, and there remayneth 3, which I write vnder the seconde place beneathe the lowest line. Then in the thirde place I finde 4, 3, 4, whiche with the 1 betweene the lines, doe make 12, therefore I write the article againe vnder the fourth place, and the digite 2

I take from 9, and there remaineth 7, which I write vnder them beneth the lowest line.

And then come I to the fourth place, where I gather 1,2,3, and the 1 betweene the lynes, ỹ maketh 7, which bycause it is but a digite, I plucke from 8, and the Remaynet is 1, and muste bee written vnder them in the fourthe place. After that come I to the fifte place, where are onely three ciphers, which make nothing, then shoulde I take that, that is to say nothing, from the figure ouer them, which is also a cipher, therefore I muste saye thus: yf I take nought from nought, there remayneth nought: so muste I write a cypher vnder them. Thē in ỹ sixt place I finde but 1, which I take out of 3 ouer him, and the Remainer is 2, that muste bee written benethe the lowest lyne in the sixt place. So go I to the seuenth, where I finde onely ciphers, and in the grosse summe ouer them a cypher also, therefore must I write their remaynet (which is nothing) with a cypher also. Then in the eyghte and laste place, I gather 1,1,2, that maketh 4, which if I take out of that 4 that is ouer them, there will nothing remayne. And that must bee noted with a cypher beneth the lowest lyne, as I haue often sayde, and so haue I ended my

worke,

woorke, and the figures stande thus.

40308964
20003428
10002342
10101461
1
00201733

Scholer. Sir, I remember you taught mee that ciphers shoulde not come in the laste place, for bycause they serue onelye to encrease the valewe of other fygures which followe them, and serue not for those fygures that go before them: and nowe in youre example you haue set two ciphers in the two laste places.

Mayster. I commende you for your remembraunce. And truth it is, I shoulde not haue set them here, but onely bycause that I would make you playnely to perceyue the arte ☞ of Subtraction. Therefore seeing that you doe nowe perceyue it, whensoeuer you shoulde write downe a cypher, looke whether any other figures bee yet behinde. And if not, then let go the cypher also, for it needeth not to write him in any latter places, where no other figure dothe followe, except it bee (as I did) to teache the vse of Sub[illegible] the playner.

Therefore

Therefore my figures muste stande thus when I haue ended my worke.

Scholer. So I woulde thinke by ÿ you taught mee before. And now I beleue I could subtract any summes.

4 0 3 0 8 9 6 4
2 0 0 0 3 4 2 8
1 0 0 0 2 3 4 2
1 0 1 0 1 4 6 1
1 1 1
2 0 1 7 3 3

Mayster. So may you, if you haue marked what I haue taught you. But bycause this thing (as all other) must be learned surely by often practise, I will propounde here two examples to you, wherein if you often exercise your selfe, you shall be ripe and perfect to subtract any other summe lightly, for in them is contained all the obseruances of whole nūber. And bicause you shall perceyue somewhat bothe howe to doe it, and also whether it bee well done when you haue prooued to doe it, therefore haue I written vnder them, bothe the Remayners.

3 0 8 9 6 4
1 0 3 1 4 5
1 0 2 5 9 7
1 0 1 0 2 4
1 1
2 1 9 8

1 2 5 6 1 4
5 4 2
6 8 1
2 0 1
1 1
1 2 4 3 9 0

Scholer. Sir, I thanke you. But I thinke I might the better doe it, if you did shewe me the working of it.

Mayster. Yea, but you muste proue youre selfe to doe some thinges that you were neuer taughte, or els you shall not be able to doe any more than you were taughte: And that were rather to learne by roate (as they call it) then by reason. And agayne there is nothing in this example or any other of whole number, but I haue taughte you the rules of them already.

Scholer. Then I truste by practise to attaine the vse of it. And is this all that I shall learne of Subtraction?

Mayst. Yea, sauing that (as you haue seen in Addition) there are numbers of diuers denominations, in which the working is not much vnlike, yet without some instructions be giue of it, it myght seeme to a learner more difficult, than in deede it is. Therefore I will briefly shewe you the vse of it onely, by one example or two.

A certaine man owed to me 14 li, 12 s, 8 d, of which hee payde mee at one tyme 4 li, 6 s, 8 d: at an other tyme 3 li: and at an other 2 li, 3 s, 4 d, and laste of all, 6 s, 8 d.

Nowe

Nowe woulde I knowe what remayneth vnpayd yet, therefore I sette my summes thus.

lb	s	d
14	12	8
4	6	8
3		
2	3	4
	6	8

Scholer. Sir, I pray you why do you write 2 lb? for the common speach vseth rather to say 40 s.

Mayster. We must heere vse the denomination that is greatest in any summe, so that wee may not write according as wee vse to speake, saying: 16 d, 18 d: or likewayes, 7 grotes, 8 grotes: 24 s, 40 s, 48 s, and such other, but we must write euery denomination that is in any summe by it selfe, namely shillings and poundes. So muste we write for these summes nowe named, 1 s, 4 d: 1 s, 6 d: 2 s, 4 d: 2 s 8 d: 1 lb 4 s: 2 lb: 2 lb, 8 s: & so forth of other like.

Scholer. So that wee may not write in Arithmetike pennies, when the summe amounteth to shillings, nor shillings, when the summe maketh poundes. Nowe if it please you, ende your example.

Mayster. When my summes are so set as I shewed, then muste I begin with the smallest denomination, saying: 8, 4, 8, are 20,

lb	ß	d
14	12	8
4	6	8
3		
2	3	4
	6	8
	1	
4	16	

which sume bycause it is pens, and 12 pens doe make 1 ß, I muste take from that 20 (whiche commeth of the 3 parcels) 12, & for them write 1 betweene the lines vnder the shillings, then the 8 d, that remayneth, I take out of the highest summe, whiche is 8 also, and then remayneth nought: wherefore vnder the pens I write nothing. Then come I to the shillings, & gather the parcels, 6,3, 6, whiche with the 1 betweene the lines, make 16, that must I take out of the summe that is ouer it. But seeing that summe is but 12, I cannot take 16 out of 12, I must borrowe one out of the 14 lb, and put to the 12, and that maketh 32, for 1 lb, is worth 20 ß: then take I 16 out of 32, and there resteth 16 to bee written vnder the shillings. Then come I to the poundes, whose parcels are 2, 3, 4, that is in all 9, and one more muste I adde thereto, bycause of the 1 that I borowed before vnto the 12 ß, and then is there 10, whiche I must take out of 14, so doth there remaine 4 to bee written vnder the poundes: so doth my remainer appeare to bee

4 lb,

4.li.16 s.

Scholer. This doe I perceyue very well, and if there be none other thing to bee learned in Subtraction, then may I come to Multiplication, for that you reckened to bee in order next.

Mayster. We haue done in deed with the arte of Subtraction, as touching the working. But yet before we go to Multiplication, I will instruct you how to examine your work whether it bee well done or no, and that is by casting away 9 as often as you can finde it, as you did in Addition, sauing that you muste heere examine the highest number alone, and note the residue of it at a lines ende, as you did in Addition.

A proofe of Subtraction in numbers of one denomination.

And when you haue done with the highest number, then examine all the other togither, casting thence 9 as often as you can: and if the last remayner bee like the other, then haue you done well.

But if you haue diuerse denominations in your summe, yet for them all shall you make but one seuerall line, as you did in Addition, remembring to begin the examination at the greatest denomination, and to double the remayner of poundes, and triple the remayner of

A proofe in Subtraction among diuers denominations.

shillings, as you did also in Addition.

As for a proofe, I will examine this worke wherin the highest line I find of pounds 14, from thence I bate 9, and there resteth 5, whiche I doe double, bycause they are pounds, and then are they 10: thereto I adde the 12, & it maketh 22, from whiche I take 9 twise, and there resteth 4, whiche bicause they are shillings, I triple, and then are they 12, thereto I adde the 8, and then are they 20, thence take I twise 9, and yet resteth 2, whiche I write at the one ende of a line thus. 2 ———

lb	s	d
14	12	8
4	6	8
3		
2	3	4
	6	8
4	16	

Then I examine all the other parcels and the remayner togither, euerye denomination by it selfe. And firste of poundes I finde 4, 3, 2, 4, that is 13, from whence I take 9, and there resteth 4, that doe I double, and it maketh 8, to it doe I put the shillings, 6, 3, 6, 16, that is 31 (for the one betweene the lines must not bee reckened, nor none in that space) and that maketh in all 39. Where hence I take 9 foure times, and there remayneth 3, that doe I take three times, and it is 9, wherefore I cast it away: then doe I take the pennies, 8, 4, 8,

that

that maketh 20, from which I take 9 twyce, and there resteth 2, which I write at the other ende of the proofe lyne. And bicause I see that those two numbers are equall, I say that I haue well wrought.

And if you will, you may make for euerye denomination a lyne, as you learned in Addition: but thē must you beginne your examination at the smallest denomination, as you did in Addition, for their proofe is altogither like, sauing that in Addition you examined the nethermoste summe alone, and all the other togither: and in Subtraction ye must examine the highest number alone, and all the other togither. And if you marke it well, it is euen all one; for that summe that in Addition is lowest, in Subtraction is highest: and that summe is called the Grosse or Totall summe.

Grosse or Totall summe.

Therefore if you marke what I sayde in Addition, you maye easilye perceyue what is to bee done for the proofe of Subtraction. And to the intent that you may perceyue it the better, I will shewe you an other proofe of Subtraction, and that shall bee by Addition, thus. Draw vnder the lowest nūber, (which is your remayner) a lyne: then adde that number, and all the other that you did subtracte before, togither

An other proofe of Subtraction.

githher, and write that that amounteth, vnder the lowest line: and if the summe that commeth thereof, bee equall to the highest of the subtraction, then was the subtractiō well wrought, or els not. As for example: in the laste summes, which stoode thus.

li.	s.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	

First I adde 8,4,8, that maketh 20, whereof I take 12 awaye, bycause they make one shilling, and write for them 1 vnder the shillings: and the 8 that is lefte, I write benethe the lowest line, then adde I the shillings 6.3.6.1.16. that make 32: from which I take 20, and for it I write 1 vnder the poundes, and the 12 that remaineth, I write vnder the shillings. Thē come I to ẏ poundes, adding them togither, which are 4.3.2.1.4. that maketh 14: then doe I write 14 vnder the li, and so haue I ended the Addition. And I see that the lowest lyne of number and the highaste bee lyke, wherefore I knowe that I haue well doone. For my figures appeare as you may playnely see in the page folowing.

And thus now haue I taughte you the arte of Subtraction, and the meanes to proue whether it bee well wroughte or not.

li.	s.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	
14	12	8

Now and you remember, I omitted in teaching the profe of Addition one waye, which I saide was by Subtraction.

Scholer. Truth it is, and then was it deferred, bycause that I had not then learned the feate of Subtraction, whereby I shoulde haue proued it, but now I thanke you, I haue well learned the arte of Subtraction, & the proues of it, bothe by 9, and by addition. And nowe I would bee glad to knowe, howe I may proue Addition by Subtraction.

Mayst. Then marke you this. When you haue ended youre Addition, take the numbers all that you did adde, to the highest summe, and deduct or subtract them from the grosse summe that dothe resulte, and if the remayner be like to the highest number, then haue you done wel, els not.

The profe of Addition by Subtraction.

As for example. I take one of the summes

that I did adde before, which was this that followeth here.

1	0	6	8	0	0
		9	4	0	0
1	1	6	2	0	0

Then doe I come to ẏ middle number (bycause here in this example are onelye three numbers) and subtract that from the nether number, beginning at the right hande, and firste I say, 0 out of 0, there remayneth 0: that write I vnder an other lyne. Thē agayne, 0 in the seconde place from 0; remayneth 0, vnder it I write 0 also. Next that in the thirde place, 4 out of 2 will not bee, therefore I adde to that 2, 10, and make it 12, from that I take 4, and there resteth 8. Then saye I farther: 9 in the fourthe place, and 1 (which I muste adde for the 10, borowed before) make 10, that must I take from 6: and bycause I canne not, I adde to the 6, 10, and then is it 16: from thence I take 10, and there resteth 6, to bee written vnder them. Agayne in the fifte place where I finde nothing written, I muste set 1 for 10 laste borowed, and that 1 do I take from the 1 vnder him, and so remaineth nought, wherfore I write downe a cipher 0. Nowe haue I done with the subtraction: & yet in the grosse summe remayneth 1, which I muste set righte in the same place, in the remayner, and so the remainer appeareth to bee like

like vnto the higheſt ſumme of the Addition, as heere appeareth. Wherefore I ſay that the Addition was well wrought. And note, that if

106800
9400
116200
106800

you had ſubtracted the vppermoſt from the product or totall ſumme, then the reſidue thereof woulde bee equall to that middlemoſte number. But if the parcels whiche you added, bee more than two: (as three, foure, fiue, ſixe, or more, (then from your groſſe or totall ſumme ſubtract firſt one of the parcelles: and note that new reſidue. Out of that newe reſidue, ſubtract an other of your parcelles, (whiche you will) and note that ſecond new reſidue. And if you haue no mo parcelles added, but three, then is that ſecond new reſidue equall and alike to the thirde parcell, whiche you haue not (as yet) ſubtracted, if you haue wrought well: both in your firſt Addition, and nowe in your ſubtracting. And ſo in this wiſe, (if you haue four, fiue, or more parcelles) maye you proceede to make your ſelfe ſure of youre totall ſumme, firſt, by Addition of the ſayde parcelles, produced and gathered. And thus may you do in any other ſumme of one denomination or many. Therefore nowe will I make an ende of

Subtra-

Subtraction, and will instruct you in Multiplication.

MVLTIPLICATION.

Multiplication is suche an operation, that by two summes produceth the thirde: whiche thirde summe so many tymes shall conteyne the first, as there are vnities in the seconde. And it serueth in the stead of many Additions. As for example. When I would know howe many are 30 times 48: if I shoulde adde 48, thirtie tymes, it would be a long worke. Therfore was this woorke of Multiplication deuised, which shall doe that at once, that Addition should do at many times,

Scholer. I perceyue the commoditie of it partly, but I shall not see the full profite of it, till I know the whole vse of it. Therefore sir I beseech you, teach me the working of it.

Multiplication of digits.

Mayster. So I iudge it best, but bicause that great summes cannot bee multiplied, but by the multiplication of digits, therefore I thinke best to shewe you first the way of multiplying them: As when I saye, 8 tymes 8, or 8 tymes

9.&c. And as for the small digits vnder 5, it were but folly to teach any rule, seeing they are so easie, that euerie childe can doe it. But for the multiplication of the greater digits, thus shall you doe.

Firste set your digittes one ouer the other righte, then from the vppermoste downwarde, & from the nethermost vpwarde, drawe straight lines, so that they make a crosse commonlye called saint Andrewes crosse, as you see herre. Then looke howe many eche of them lacketh of 10, and write that vnder eche of them, at the ende of the lines, and that is called the Differences: as if I would know how many are 7 times 8, I must write those digits thus.

The difference.

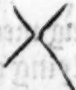
Digit. Differen.

8

7

Then do I looke how much 8 dothe differ from 10, and I finde it to bee 2, that 2 doe I write at the right hande of 8, at the ende of the line, thus.

8 2



7

After that I take the difference of 7 likewise from 10, that is 3, and I write that at the right side of 7, as you see in this example.

8 2

7 3

Then do I drawe a line vnder them, as in Addition, thus.

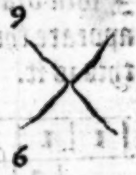
Last of all I multiply the two differences, saying: 2 times 3 make 6, that must I euer set vnder the differences, beneath the lyne: then muste I take the one of the differences (which I will, for all is like) from the other digit (not from his owne) as the lines of the crosse warne mee, and that ỷ is left must I write vnder the digits. As in this example. If I take 2 from 7, or 3 from 8, there remayneth 5: ỷ 5 muste I write vnder ỷ digits: & thẽ there appeareth the multiplication of 7 times 8, to be 56. And so likewayes of any other digittes, if they bee aboue 5: for if they be vnder 5, then will their differences bee greater than themselfe, so that they can not be taken out of them. And againe, such little summes euery childe can multiply, as to say: 2 tymes 3, or 4 times 5, and such like.

Digit.	*Differen.*
8	2
7	3
5	6

Scholer. Truth it is. And seeing mee seemeth that I vnderstand the multiplying of the greater digittes, I will proue by an example how I can doe it. I would knowe how many are 9 times 6.

Mayster. It is all one in value to saye 9 times 6, or 6 times 9: but yet the order is best

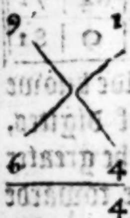
to put the lesse summe firste, saying: 6 tymes 9; and so of all other summes.



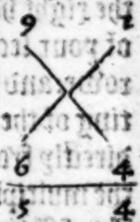
Schol. Then would I know, howe many are 6 times 9: therefore I set the digites thus, and make the crosse thus.

Then doe I set their differences at the right side: the difference of 9 which is 1 against it, and the difference of 6, which is 4 against it also, as in this example.

And vnder them I drawe a line. Then doe I multiplye the digites togither, saying: one time 4, maketh 4, that 4 doe I write vnder the differences thus.



Then take I one of the differences from the other digite, as one from 6, or else 4 from 9, and eche wayes there resteth 5, which I do write vnder the digites thus. And so appeareth the multiplication of 6 tymes 9, to bee 54. Thus I see the feate of this maner of multiplication of digits.



Mayster. Nowe might you go straight to the multiplication of greater numbers, saue yͤ

bothe for youre ease and suretie in working, I will draw you here a table, whereby shall appeare the multiplication of all digites, and this is it.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which figure, when you woulde knowe the producte in any multiplication of digites, seeke youre firste or laste digite in the greater figures, and from it go right forthe towarde the right hand, tyll you come vnder the number of your seconde digite, which is in the highest rowe: and then the number that is in the meeting of the rowes of little squares (which come directly from bothe your propounded digits) is the multiplication that amounteth of them. As yf I woulde knowe by this table the multiplication of 7 times 9, seeke first 7 in ẏ greater

ter

ter figures, and then go right forthe towardes the right hande, tyll you come vnder 9 of the highest rowe, in which place, where you so come vnder the other digitte (as here for example you come vnder 9) is alwaies contayned the ofcome or producte, which you seeke: and that place wee terme to bee in the common angle, in respect of the two numbers so taken on the outsides, as here in that common angle, where the rewes of little squares (directly proceding from 7 and 9) do meete, you haue 63: which 63, is the summe of the multiplication of 9 by 7.

Scholer. This is very good and readye. And so may I finde the multiplication of any digittes. But nowe howe shall I do in greater summes?

Mayster. When you woulde multiply any summe by an other, you shall marke that it is the meetest order to sette the greatest number highest, which is the place of the number that must bee multiplyed: and likewaies the lesser number vnder it, for that is the place of the Multiplyer or Multiplicatour, that is to say, ẏ number by which multiplication is made: and is in Englishe alwayes put before this worde, Tymes: in suche speaking when I saye, 20

times 70. And the number that followeth this worde Tymes, is that which muste bee multiplyed.

Therefore when I would multiply one nūber by an other, I muste write the greatest highest, and the lesser vnder it, as in Addition: And vnder them muste I drawe a lyne. As for example: If I would multiply 264 by 29, I must set them thus.

2 6 4
2 9

Then must I multiply euery figure of the higher rowe, by euery figure of the nether rowe: and that that amounteth, I muste set vnder the lyne, as thus. Firste I doe multiplye 4 by 9, saying: 9 times 4 (or 4 tymes 9, which is all one) and that maketh 36, as ẏ table before of digittes wthe declare: of that 36 I muste write the 6 that is the digitte, vnder the 9, and the 3 in the next place towarde the left hande.

2 6 4
2 9
3 6

Then come I to the ij. figure of the higher rowe, and say: 9 tymes 6 make 54, of which I write the 4 vnder the 3, and the 5 vnder the next place (as the reason willeth mee) thus.

2 6 4
2 9
5 3 6
4

After that come I to the next figure, which is 2, and wil mul-

tiply

tiply it by 9, and that maketh 18: whereof I write 8 vnder the thirde place, and the article 1 in the fourth place, thus.

	2	6	4
		2	9
1	5	3	6
	8	4	

And so haue I ended the first figure of the multiplier. Wherfore I giue it now a fine dashe with my pen.

Then beginne I with the nexte figure, and multiplie it into all the higher figures, as thus. First 2 times 4, make 8, that do I write vnder ẏ second place: for euermore the digitte or firste figure of the multiplication that

	2	6	4
		2	9
1	5	3	6
	8	4	
	8		

amounteth of the firste figure of the higher nūber, mast bee set vnder the multiplier of it, and the other in their order, towarde the left hand.

Scholer. I vnderstande you thus: that the digit of the summe amounting of the multiplication of the first figure of the higher rowe, by the first figure of the lower rowe or multiplier, must be set vnder the first place: and that that amounteth of the same firste figure by the seconde multiplyer, must bee set vnder the seconde place: & so of the other, if there be more multipliers.

Mayster. So meante I in dede: and if

there amount but a digit, then muste it bee set vnder the multiplier.

And now to go foorth: I multiplye by the same 2, the seconde figure of the higher rowe, whiche is 6, saying: 2 tymes 6, make 12: whereof I write the digit 2 vnder the third place and the article 1, I write vnder the fourth place.

```
 264
  28
----
1536
184
```

Then do I multiplie the last figure of the highest summe, by that same 2, saying: 2 tymes 2 is 4: whiche I write vnder the fourth place. And so haue I ended the whole multiplication: wherfore I also giue the a dash with my pen, thus: and so I do euer assone as I haue dispatched any diget by whiche I multiplie. And the summes stand thus.

```
  28
```

```
 264
  28
----
1536
184
428
```

Then must I drawe a line vnder all those summes that amounte of the multiplication, and must adde all them into one summe, as in this example you may see.

```
 264
  28
----
1536
184
428
----
7656
```

Where in the first place I finde but 6, and therefore write I it vnder the line. Then in the seconde place

place 8,4,3, make 15, whereof I write 5, and keepe one in my minde, and ſo forth, as you learned in Addition. And ſo appeareth the whole ſumme to bee 7656, which amounteth of the multiplication of 264 by 29.

Scholer. If there be no more to be obſerued in it, then can I do it, I ſuppoſe, as by this example I ſhall proue. I would multiplie 1365, by 236, wherefore I ſet them thus.

$$\begin{array}{r} 1365 \\ 236 \\ \hline \end{array}$$

Then doe I multiplye 5 by 6, ſaying: 6 tymes 5 make 30: of whiche I write the cipher in the firſt place, and the article 3 in the ſeconde place.

$$\begin{array}{r} 1365 \\ 236 \\ \hline 30 \end{array}$$

Then do I by the ſame 6, multiplie the ſeconde figure of the higher ſumme, whiche is 6, ſaying: 6 tymes 6, make 36: of whiche I write the 6 vnder the ſeconde place, and the 3 vnder the thirde place

$$\begin{array}{r} 1365 \\ 236 \\ \hline 330 \\ 6 \end{array}$$

Then doe I multiple the thirde figure whiche is 3, by the ſame 6, and that maketh 18: of that I ſet the 8 vnder the thirde place, and the 1 in the fourth place.

$$\begin{array}{r} 1365 \\ 236 \\ \hline 1330 \\ 86 \end{array}$$

Then come I to the last figure of the higher summe, and multiplie it by 6, saying: 6 tymes 1 make 6: that doe I write vnder the fourthe place.

1365
236
1330
686

And so haue I ended the last multiplier, and dashe him slightly wt my pen.

Then begin I with the seconde multiplier, and say first 3 times 5, that maketh 15, of whiche I sette the 5 vnder the seconde place, bicause that the multiplier is there, and the 1 I set vnder the thirde place.

1365
236
1330
686
15

Then come I to the seconde figure that is 6, and multiplie it by 3, which maketh 18, of whiche I set the 8 vnder the thirde place, and the article 1 in the fourth place.

1365
236
1330
686
215
8

Then come I to the thirde figure, whiche is 3, and multiply it by 3, saying: 3 times 3, make 9, whiche bicause it is but one digit, I set vnder the fourth place.

1365
236
1330
686
215
98

And then comming to the

laste

laste figure 1, I multiplye it by 3, and it maketh 3, whiche I set in the fift place, and then haue I ended two of the multipliers, and ẏ summes stande thus. And then I geue 3 his dashe.

```
 1 3 6 5
   2 3 6
 1 3 3 0
 6 8 6
 1 1 5
3 9 8
```

```
 1 3 6 5
   2 3 6
 1 3 3 0
 6 8 6
 1 1 5
3 9 8
  1 0
```

Then come I to the thirde multiplyer, and multiply it into euery figure of ẏ higher summe, and firste I saye: 2 times 5 makes 10, of whiche I sette the cipher vnder the multiplier in the thirde place, and the article 1 in the fourth place.

```
 1 3 6 5
   2 3 6
 1 3 3 0
 6 8 6
 1 1 5
3 9 8
 1 1 0
 2
```

And so multiplying the seconde figure 6 by that same 2, there amounteth 12: whereof I write the digitte 2, vnder the fourth place, & the article 1, vnder in the fifte place.

Nowe we I multiplye by the thirde figure of the higher summe, whiche is 3, and that maketh 6: whiche I sette vnder the fifte place, as appeareth in the page folowing.

1365
236
1330
686
115
398
110
62

Then come I to the laste place, and multiplye that by 2, and there amounteth 2, which I sette in the firste place, & then be the summes stande thus.

1365
236
1330
686
115
398
110
262
322140

And so haue I ended the whole multiplication.

But now (as you taught mee) to knowe what this whole summe is, I muste adde all those parcelles togither, and then vnder the line will appeare as you see, the grosse or totall summe, that is, 322140.

Maister. That is well done.

Scholer. Then mee thinketh I would call it well done, when I knewe whether I had well done or no.

Mayster. It may bee tried by 9, as addition was, but the surest proofe is by Diuision, and therefore I will reserue that tyll you haue learned the arte of Diuision.

And before wee passe from Multiplication,

I will yet shew you other waies of Multiplication, which are counted of some men bothe more ready and more certaine, of which the one differeth nothing from this that I haue taught you, saue that it dothe vnderstande alwaies the articles, and ioyne them to the next summe, and therefore I will declare it onelye by an example.

If I woulde multiplye 1542, by 365, I must set them as I saide before, and then doe I multiply 2 by 5, and it maketh 10, of which I write the article vnder the first place, and keepe the digit 1 in my minde.

```
1542
 365
----
   0
```

Then say I forthe: 5 times 4 doe make 20, and the 1 in my minde, are 21, thereof I write the 1 vnder the seconde place, and keepe the 2 in my minde.

```
1542
 365
----
  10
```

Then come I to the thirde figure 5, saying: 5 times 5, make 25, and the 2 in my minde, make 27, whereof I write the 7 vnder the third place, and keepe the article 2 in my minde.

```
1542
 365
----
 710
```

Then comming to the last figure,

I say: 5 tymes 1 make 5, and 2 in my minde make 7: that do I write vnder the fourth place.

1542
365
7710

And then haue I ended my first multiplier, and therfore I dashe it.

Then do I likewaies with the seconde multiplyer, saying: 6 tymes 2 make 12, thereof I write the digit 2 vnder the seconde place, and keepe the article 1 in my minde.

1542
365
7710
2

Then say I forth: 6 times 4 maketh 24, and 1 in my minde make 25, so I set that 5 vnder the thirde place, and keepe the 2 in my minde.

1542
365
7710
52

Then multiply I forth, saying: 6 tymes 5, maketh 30, and 2 in minde make 32, whereof I write the 2 vnder the iiij. place, and keepe the 3 in my minde.

1542
365
7710
252

Then do I multiply the laste figure 1 by 6, and it maketh 6, to that I adde the 3 in my minde, and it maketh 9, which I write in the fift place.

1542
365
7710
9252

And so haue I ended two figures of the multiplyer.

Then with the thirde and

laste multiplier, doe I likewayes, and saye firste: 3 times 2, make 6: whiche I write in the third place vnder the multiplier. Then by

1542
365
7710
9252
6

1542
365
7710
9252
26

that 3 doe I multiplie likewayes the seconde figure 4, & it maketh 12, wherof I write the digit 2 vnder the fourth place, and the article 1 I keepe in minde.

Then come I to the thirde figure 5, saying: 3 times 5 maketh 15, and the 1 in my minde make 16, thereof I write the 6 vnder ye fift place, and keepe the article 1 in my minde.

1542
365
7710
9252
626

Then come I to the last figure which is 1, and multiplie it by 3, and it maketh 3, thereto I adde the 1 in my minde, and it maketh 4, which I write in the 6 place. And then haue I ended the multiplication, and the figures stande in order thus.

1542
365
7710
9252
4626

Which parcels if I adde into one summe,

it will be 5 6 2 8 3 0, whiche is the grosse or totall summe of all that multiplication.

Scholer. Well, this maner of Multiplication I perceyue: but what other sortes haue you?

Mayster. There is one way that is wrought by a checker table, made thus.

An other way of Multiplication.

Looke howe many places your summe hath that you woulde multiply, so many squares muste you make in your table, from the right syde to the lefte: and so manie from the higher parte to the lower, as there be places in your multiplyer. Then set downe your greatest summe, first on the toppe of the table, euerie figure in due order, in a square alone, I meane in those squares that bee open & vncrossed. And likewise in those like squares at the right hand, set downe your multiplicatour or multiplier, the last figure in the highest place, and so downewarde, that the first figure may be in the lowest place.

Scholer. Sir if it please you, me thinketh then I vnderstand you best, when you doe not stand long in telling the rule before examples: But propose some example, and then in declaring

cing it, bꝛing in the rules withall.

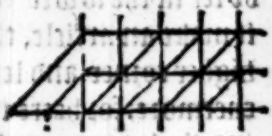
Mayster. In deede, that way is easiest foꝛ a yong learner, therfoꝛe will I euen do so. Take this example: nowe I would multiplie 2036 by 25.

First I consider that my greatest number hath foure figures oꝛ places, & therefoꝛe I make so manye roumes betweene lines, thus.

Then I see that of my multiplyers there are two, wherefoꝛe I dꝛawe so manye lines a crosse the other, that there may bee two roumes betweene them, thus.

But you muste not foꝛget to let the endes of the lines runne out, as it appeareth in this Patrō, foꝛ in those open squares must your two first numbers, and all the totall summe be set.

Then dꝛawe a crosse bar thꝛough euery close square, so that it may reach down to the lowest ouerthwart lyne, as in this foꝛme. And then is your checker foꝛme pꝛepared.



Then sette downe youre first oꝛ greatest summe

summe on the toppe, And your multiplier on the right side in the open squares thus.

Then begin to multiplie the first figure of the higher summe, by the highest of the multiplier, saying: 2 times 6 make 12, that 12 must you write in the square that is agaynst the 2 and the 6, but in suche manner that the digitte bee set in the nether corner of the square, and the article in the higher corner: as you may see in this example.

And so of every other multiplication, what ever amounteth you must write in the common square, whiche is agaynst bothe those figures, by which you do multiplie. And if that summe doe make but one digitte, then must it be set in the lower corner of the square, but if it make an article, then write the article in the higher corner, and let the cipher go (if you will) evermore: for heere it serveth for nothing, seeing the lines doe distinct the places: but if the summe amounting of such multiplication doe make a mixt number, then write the article in the higher corner, and the digitte in the lower

corner,

corner, as I did by that 12.

Then when you haue multiplyed and ended the firste figure, come to the nexte, and multiply it in like manner, as in saying: 2 tymes 3, is 6: that 6, bicause it is but a digit, you shall set in the nether corner of the square, next vnder 3, thus.

Then go forthe, saying: 2 tymes 0 is 0: set that vnder the barre (if you liste) in the thirde square.

Then forthe and saye: 2 tymes 2 make 4, that set in ye laste square vnder the barre, so haue you ended the firste multiplyer: Dashe him.

Come nowe to the seconde multiplyer, and say: 3 tymes 6, make 18, of which summe, ye article 1 muste be set aboue the barre, in the square that is next to ye 3, (as you se here) and the 8 vnder the barre.

Then say: 3 tymes 3, make 9, set it in the next square benethe the barre. Then 3 tymes 0 is 0, write it in the nexte square, or let it go, for all is one.

Scholer. I perceyue it well: for here the

lines

lines distincte the places, wherefore ciphers doe onely serue, and therefore here they neede not to be.

Mayster. Then say farther: 3 tymes 2, make 6: write that in the laste square, then will the whole figure stande thus.

Sch. Now could I (me seemeth) doe like againe. But how shall I doe now to gather the summe?

Mayster. Marke firste the order of the places in this figure, and so shall you perceyue the reason of gathering them into a summe.

The slope barres doe part the places, so that the first place is ỹ lowest corner (in all such figures) of the nethermost square next the right hand: and all the halfe squares betweene that barre and the next, standeth for the seconde place, and so the roome betweene that and the next barre, is the third place: & so forth. Now if you perceyue this, then muste you adde all the figures of one place togither, as if you had an Addition of diuers summes.

Scholer. If I vnderstande you right, then must I take here in this example 8 to be in the firste place: 9, 1, and 2, in the seconde: 1, 0, 6, 3, in the third: 6, 0, in the fourth: 4 in the fifte: and the

the first place hath no figure.

Maister. You say well, and the reason is bicause the multiplication seruing to ẏ square, made but a digit.

Scholer. Then it is all one, as if they stoode thus.

		5	1	2
		0	6	1
4	6	0	9	8

Mayster. Euen so it is: and now adde this summe, and there will appeare the totall of the multiplication to bee, 46828.

And if you will see the agreement of this manner of Multiplication, and the other that you learned before, then multiplye those two summes (that is 2036, by 23) after the firste manner without squares. A proofe vvithout squares.

2	0	3	6
		2	3

Scholer. You meane to set them thus in order.

And then multiply 3 into 6 make 18: 3 times 3, make 9, 5 tymes 0 is 0, then 3 tymes 2 make 6: which must be set thus.

2	0	3	6
		2	3
		1	8
6	0	9	

Then doe I likewaies with ẏ seconde multiplyer, saying: 2 tymes 6 make 12, 2 tymes 3 make 6, 2 tymes 0 is 0, and 2 tymes 2 make 4, which when I adde to the other, then wyll

the whole multiplication stãd thus.

```
  2 0 3 6
      2 3
  -------
      1 8
    6 0 9
      1 2
  4 0 6
  -------
4 6 8 2 8
```

Mayster. So that you may see in euery place the same figures, as they were in ye multiplication by squares, though they differ in height and lownesse of places, but being added togither, they make one summe.

And thus nowe ye haue learned three sortes of multiplication, which you like beste, that may you vse.

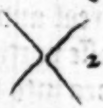
Yet are there other formes, but sith they nothing differ from these three in effect, but onely in setting of the numbers, I will ouerpasse them till a more meeter place and tyme. And nowe will I instruct you in Diuision, so that you thinke your selfe sufficientlye to perceyue what I haue taught you.

Scholer. Yes sir I thanke you, but I doe not perceyue howe to examine my worke, to try whether I haue well done or no.

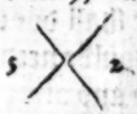
Mayster. That is commonly vsed by the proofe of 9, as you learned before in Addition and Subtraction, saue that it hath this waies diuers from them.

First you must make a crosse after this maner.

Then muste you examine youre summe that shoulde bee multiplied, and looke what remayneth after casting away of 9, that set you at the one side of the crosse: then examine the multiplyer, and whatsoeuer remayneth in it, after casting away 9 as often as you can, write that at the other side of the crosse: then muste you multiplie those two numbers togither, and looke what amounteth thereof, if it be vnder 9, write it at the higher part of the crosse: but if it bee aboue 9, then take thence 9 as often as you can, and write the rest at the head of the crosse. As in the laste example of multiplication, the number to bee multiplyed is 2036, wherein is once 9, and 2 remayneth, which I write at one side of a crosse, thus.



Then do I examine the multiplier, which is 23, wherein there is no 9, but 5 in all, that 5 therefore I set at the other side of the crosse, thus.



Then doe I multiplie 5 by 2, and it maketh 10, from whiche I withdraw 9, and there resteth 1, that 1 doe I set at the head of the crosse, then doe I examine the grosse

summe amounting of the multiplication, whiche is 46828, wherein, I finde 9 three tymes and 1 remayning, that 1 I set at the foote of the crosse, and then I see it to agree with the other 1 at the top of the crosse, and so know I that I haue done well: for if they two did differ, then were my worke vaine, and the multiplication false.

This is the common proofe, but the moste certaine proofe is by Diuision, of which I will anon instruct you.

Scholer. Syr, what is the chiefe vse of Multiplication?

Mayster. The vse of it is greater than you can yet vnderstand: howbeit, these plaine commodities it hathe; that if you woulde resolue any great and whole valure into many small and lesse portions: as if you woulde chaunge poundes into shillings, pennies or any other greater or smaller parcels, by multiplication, ye shall doe it speedily and easily. Also if you shoulde neede to adde one summe to it selfe, or to any other oftentymes, you shall doe it by Multiplication muche more speedily, readilye, easilye and surely, than by often and sundrie Additions. Take you these commodities

grossely

grossely shewed for an answere at this tyme, and hereafter I will more abundantly make you to perceyue the vse of it.

DIVISION.

Scholer.

WEll sir, then in Diuision I praye you to instructe mee. But mee thinketh by the name of it, that it shoulde bee all one with Multiplication: for I call that Diuision, when anye thing is parted into dyuers and manye partes.

Mayster. You take it as it is taken commonlye, howbeit, if you marke well, you shall perceyue that it is quite contrarie to Multiplication, and dothe not part one thing or fewe things into many, but contrary wayes it bringeth manye parcels into fewe, but yet so, that these fewe taken togither, are equall in valure to the other many: for by Diuision pennies are turned into shillinges, and shillinges into poundes: as for example of 120 shillinges, it

maketh 6 pounde, so are 120 turned into 6, which is a smaller number: but then if you consider the denominatours, you shall see that they are such, that one of the latter is equall to 20 of the first, and so in value the summes are one, though in number they do farre differ, and the latter summe is the lesser, and so is it alwayes in diuision, how be it, yet in the working, the summe is parted by an other, & thereof doth it take the name.

Scholer. I thinke I shall better vnderstande the reason of the name, when I knowe the vse of the worke, therefore nowe woulde I gladly learne that.

Diuision what it is.

Mayster. Diuision is a distributing of a greater summe by the vnities of a lesser. Or Diuision is an Arithmeticall producing of a thirde number, in respect of two propounded numbers: which thirde number shall so often conteyne an vnit, as the greater of the twoo propounded numbers doth containe the lesser. So that, euen as Multiplication did seeme to serue in stead of manye Additions, so Diuision may seeme to bee in place of many Subtractions: Bicause that thirde number briefly expresseth, howe many tymes the lesser of your two propounde numbers may be Subtracted,

from

from yͤ greater: As in practiſe will more plainly appeare. Therefore (as you may perceyue) vnto Diuiſion are required three numbers: the firſte, which ſhould bee diuided, and that muſte (generally) bee the greater: and the ſeconde, by which the other muſte bee diuided, and that is (generally) the leſſer, and is called the Diuiſor. And the thirde which aunſwereth to the queſtion, How many tymes: and therefore is called the Quotient.

A generall rule for placing the figures.

The firſte muſt be firſte written, and the ſecond ſo ſet vnder it, that the laſte figure of the lower number bee ryght vnder the laſte of the higher, contrary wayes to the worke of the other kindes of Arithmetike: for in them the two firſt figures were ſet euer meete one vnder yͤ other, but in Diuiſion the laſte figures muſte bee ſet meete, except it chaunce ſo, that the laſte figure of yͤ Diuiſor bee greater than the laſte of the higher number, for then you ſhall ſet the laſte of the Diuiſor, vnder the laſt (ſaue one) of the higher number, as for example.

An exception.

If you ſhoulde diuide 365 (which are the ſumme of the dayes of a yeare) by 28, which are the dayes of a common moneth, then ſhoulde you ſet them thus.

3	6	5
2	8	

 But

But if you would diuide those 365 dayes, by 52, which is the number of weekes in one yeare, then should you set them thus.

3 6 5
5 2

Likewayes if I woulde diuide the same 365 by 4, which is the summe of the quarters of a yeare, then muste I set them thus.

3 6 5
4

Scholer. Syr, this doe I understande, but howe nowe shoulde I doe to diuide the one by the other?

Mayster. You muste beginne with the laste figure next the left hande, and see howe many times the laste figure of the diuisor may be taken out of the laste figure of the ouer number, and that shall you note within a crooked lyne toward your right hande. As for example.

I woulde diuide 365 by 28, then set I those two summes thus.

3 6 5
2 8

And I looke how many times I may finde 2 (which is the laste figure of the diuisor) in 3, (which is the laste of the number to bee diuided) and considering that I can take 2 out of 3 but once, I make a crooked lyne at the right hande of the numbers, & within it

in it

in it I set 1, and that is called the Quotient number, as I tolde you. Then bycause that when 2 is taken out of 3, there remaineth 1, I must write that 1 ouer 3, and deface or cancell the 3 and the 2, then will the figures stande thus. Quotient number.

Then muste I go to the nexte figure of the diuisor, and take it likewayes so many tymes out of the figures that bee ouer it, and loke what doth remayne, that I muste write ouer them, and cancell them, as in this example.

Therefore nowe I take once 8 out of 16, and there remayneth 8, which I muste set ouer the 6, and cancell or crosse out the 16, and the 8 of the diuisor: And then will the figures stande thus. And so haue I once wrought.

Scholer. So I perceyue that you take the nether figure not onely out of the other that is ryght ouer him, but out of that with the other also that remayneth before, and are written towarde the left hande.

Mayster. So muste you doe: for you muste so take the diuisor out of the ouer number, that there remaine not ouer it so great a summe as

it selfe is, for then were your worke in vayne.

But yet agayne here muste you marke, that when you seeke howe many tymes the laste figure of the diuisor may bee founde in the number ouer him, that you looke also whether you may as often finde all the figures following in those that are aboue them, (considering all the remainders if there bee any) if not, take youre Quotient lesse by one, and then proue againe, & so still, till you finde a meete Quotient: And by that meete quotient must you alwaies multiply your diuisor, and the product sette vnder your diuisor, so that his first figure stande vnder the firste figure of your diuisor, and the seconde vnder the seconde, and so forth: and then subtract that product from the number to be diuided, that standeth directly ouer it, as you haue seene mee doe.

When you haue thus wrought once, then must you begin againe, and write your diuisour a new, nearer towarde the right hande by one place, as in this example, you shall sette 2 vnder 8, and 8 vnder 5, thus.

```
  2
1 8 6
3 6 5 ( 13
2 8 8
  2
```

Then (as before) seeke how many tymes you may take your diuisor out of the

the number ouer him now.

Scholer. That may I do here 4 tymes.

Mayster. Truth it is that you maye finde 2, foure times in 8 : but then marke whether you can finde the figure following so manye times in the other that is ouer him. Can you finde 8 foure times in 5 ?

Scholer. No, neither yet once.

Mayster. Therefore take 2 oute of 8, once lesse.

Scholer. That is 3 times.

Mayster. Well, then 3 times 2 make 6: if I take 6 out of 8, there remayneth 2 : which 2 with the 5 following, make 25, in whiche summe I maye finde 8 iij. times also, and therefore I take 3 as true quotient, and write it within the crooked line of the quotient, before the one, thus.

Marke hovve to consider thiskinde of remaine.

```
  2
1 8 (1
3 6 5   (13
2 8 8
  2
```

Then say I : 3 times 2 make 6, then 6 out of 8, resteth 2, therefore I cancell the 2 and the 8, and write ouer it the 2 ye doth remaine, thus.

```
  2
1 8
3 6 5   (13
2 8 8
  2
```

Then doe I take 8 as many times out of 25, saying: 3 times 8 make 24, and if I take 24

out

out of 25, there remayneth 1: so then I cancell 25 and 8, and ouer the 5 I set 1, thus.

```
2
2 8 1
3 6 5 (13
2 8 8
  2
```

Or you mought (after you had found three to be a fit quotient) straight way haue multiplyed the whole diuisor 28, by that 3, at once: whiche giueth 84, whiche beeing set vnder 28, and duely subtracted from 85, of the number diuidend, giueth 1, the remayner of the whole diuisiō: as before you had, worke which way you list: here you see also ẏ forme.

```
  8 1
3 6 5 (13
2 8 8
  2 4
  8
```

And now haue I done with diuiding, for I can finde my diuisour 28 no more in the ouer summe.

Scholer. No, except you woulde part the 1 that remayneth into 28 partes.

Mayster. That is well sayde, and so must we doe in suche cases, when there remaineth any thing: but I will let that passe now, & will make you perfecte in diuision of whole numbers, and will hereafter teache you peculiarly of broken numbers, called Fractions.

Now if you do perceyue the order of diuision, then doe you diuide this summe, 13628.

by 453.

Scholer. First I sette downe the number that should be diuided, then doe I set the diuisor vnder it, so that the last figure of it be right vnder the laste figure of the ouer number. Then will it be thus.

1 3 6 2 8 0
4 5 3

Mayster. Can you take the laste of your diuisor (which is 4) out of 1, which is the last of the ouer number?

Scholer. I had forgotten, bicause the laste of the diuisor cannot bee taken out of the laste of the ouer number, in as much as it is the greater, therfore must I set the diuisor one place more forwarde towarde the right hande, thus.

1 3 6 2 8 0
4 5 3

And then must I looke howe often I maye finde the last figure of the diuisor (that is 4) in 13, whiche thing I may doe 3 times, therefore doe I saye: 3 times 4 is 12, whiche I take out of 13, and there remayneth 1. Then doe I make at the right hande of my summes a croked line, and write before it my quotient 3: and I cancell 13 and 4, and ouer the 3 I set the 1 that remayneth, and then the figures stande thus.

1
~~1~~ ~~3~~ 6 2 8 0 (3
~~4~~ 5 3

Then

Then do I multiplie the same quotient into euery figure of the diuisor, and withdrawe the summe that amounteth out of the numbers ouer them, as firste I say: 3 times 5, make 15, whiche I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remayneth, thus.

```
   ~~1~~ 1
~~1~~ ~~3~~ ~~6~~ 2 6 0 (3
   ~~4~~ ~~5~~ 2
```

Then doe I say lykewayes, 3 times 2 makes 6, whiche I take out of 12, and there resteth 6, therefore I cancell the 12 and the 2, and ouer the 2 I write 6 that remayneth, thus.

```
   ~~1~~ ~~1~~ 6
~~1~~ ~~3~~ ~~6~~ ~~2~~ 8 0 (3
   ~~4~~ ~~5~~ ~~2~~
```

Then shoulde I set forwarde the diuisor, into the next place toward the right hande, thus.

```
   ~~1~~ ~~1~~ 6
~~1~~ ~~3~~ ~~6~~ ~~2~~ 8 0 (3
   ~~4~~ ~~5~~ ~~2~~ 2
          4 5
```

Mayster. But you may see, that ouer the 4 is no figure, therefore must I set the diuisor yet forewarder by an other place.

And marke, when so euer it chaunceth so, that you should set forwarde the diuisor, and that it can not stande there, bicause there is no number ouer the laste place, or if there bee any, it is lesser

lesser than the laste figure of the diuisor, then must you remoue the diuisor yet once agayne: and bicause that his firste place of remouing serued not to subtracte him so much as once, therefore shall you write in the quotient a cypher o. And if you shoulde by chaunce neede to doe so often times, for euerytime write a cipher in the quotient. The reason of this, will I shew you hereafter.

Scholer. Then must I set my summes thus.

$$\begin{array}{llllll} \cancel{1} & \cancel{1} & 6 & & & \\ \cancel{1} & \cancel{3} & \cancel{6} & \cancel{2} & 8 & 0 \quad (30 \\ \cancel{4} & \cancel{5} & \cancel{2} & 5 & 2 & \\ & & 4 & & & \end{array}$$

And bycause I remoued the diuisor, so that I ouerskipped one place, I muste write a cipher in the quotient: & then muste I seeke a new quotient, as in this example I must say, How many times 4 is there in 6? and sith it can bee but once, therefore doe I write 1 in the quotient, & then say I: 1 time 4, taken out of 6, remayneth 2, I cancell the 6 and the 4, & write 2 ouer them thus.

$$\begin{array}{llllll} & & 2 & & & \\ \cancel{1} & \cancel{1} & 6 & & & \\ \cancel{1} & \cancel{3} & \cancel{6} & \cancel{2} & 8 & 0 \quad (301 \\ 4 & \cancel{5} & \cancel{2} & 5 & 2 & \\ & & 4 & & & \end{array}$$

$$\begin{array}{llllll} & & 2 & & & \\ \cancel{1} & \cancel{1} & \cancel{6} & & & \\ \cancel{1} & \cancel{3} & \cancel{6} & \cancel{2} & 8 & 0 \quad (301 \\ \cancel{4} & \cancel{5} & \cancel{2} & 5 & 2 & \\ & & \cancel{4} & & & \end{array}$$

Then saye I agayne, once 5 out of 28, remay-

neth

neth 23, I let the 2 stande as it did, and ouer the 8 I set 3, cancelling the 8 and the 5 vnder it, thus.

```
        2
  1 1 6 3
1 3 6 2 8 0 (301
  4 5 2 5 2
      4
```

Mayster. You might as well haue [illegible]de, once 5 out of 8, and [illegible] remaineth 3, but now go forthe.

Scholer. Then once 2 out of 0, can not bee, what shall I now doe?

Mayster. Borow of the next number that is behinde (for there is 230) and doe as you learned in Subtraction in a like case.

Scholer. Then muste I borowe 1 of the 3 comming behinde nexte, and make that 0 to bee 10: and then take I 2 out of 10, & there resteth 8. And bicause I borowed one of the 3, I must cancell the 3, and write 2 ouer it: then doth the figure stande thus.

```
      2 2
  1 1 6 3 8
1 3 6 2 8 0 (301
  4 5 2 5 2
      4
```

Mayster. Now haue you done, and yet remayneth 228, and your quotient doth shew you, that if you diuide 136280 by 452, you shall finde your diuisour in youre greater number 301, that is CCC. tymes, and once 228 remayning.

And in the other example, where I diuided

365 by 28, the quotient was 13, and 1 remayned: whereby I know that in a yeare (which contayneth 365 dayes) there are 13 monethes, reckening 28 dayes (or 4 weekes) iuste to a moneth, and 1 daye more.

Scholer. Why then we wee call a yeare but 12 monethes?

Maister. Of that at a more conuenient tyme wyll I fullye instructe you: but nowe it is not conuenient to entangle your minde with other things, than we directlye pertayne to your matter. Therefore if you can remember what you haue hearde, you haue learned a short mãner of diuision, which I woulde haue you often to practise, so that you may be perfect in it, and hereafter I will shewe you certaine other proper poyntes touching it.

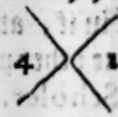
Scholer. Then I pray you, yet tell mee, how I shall examine and trye my worke, whether I haue done well or no, that though no man be by mee to tell mee, yet I may perceyue it my selfe.

Mayster. Some men (yea and commonly) we trie it by the rule of 9, as in all the other kindes, saue that their order is this. Firste they caste away 9, as often as they can, out of the diuisor, and that that remayneth, they set at Proofe.

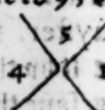
 one

one side of a crosse: As in our first example, the diuisor was 28, from which you may take 9 three times, & 1 remaineth, which they set by a cros, thus.

Then doe they likewise examine the quotient (which in our example is 13) and from thence they caste away 9 as often as they can, and the remainer they set at the other side of the crosse; and then multiply they togither those ij. remainers: and to it that amounteth they adde the remainer of the diuision, if there were any, from that whole summe they withdrawe 9 as often as they can, and the rest they set at the heade of the crosse: as in our example the quotient is 13, from which take 9, and there remaineth onely 4, and therfore must you set 4 at the other side of the crosse, thus.



Then multiply 4 by 1, and it yeldeth but 4, thereto adde the remayner of the diuision (which was 1) and it will bee 5, which summe doth not amounte to 9, & therefore must bee set wholy at the heade of the crosse, as you see here.



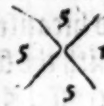
And this number on the heade of the crosse, is the first proofe, to which if you

finde

finde all other like in the number that was diuided, then haue you done well.

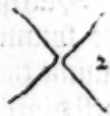
Therefore nowe shall you likewise examine the whole summe that was diuided, and take away 9 as often as you can, and that that remayneth, set at the foote of the crosse: and if it be equall to that in the head of the crosse, then haue you well done, else not.

As in our example the whole summe was 365, which maketh 14, from that take 9, and there resteth 5, which set at the foot of ÿ crosse, thus.

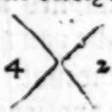


And you shall see that they agree: therfore haue you well done.

Scholer. Nowe will I likewise examine our second example, where the diuisor was 452 whiche maketh 11: from thence I take 9, and the 2 that remayneth I sette at the right side of a crosse, thus.

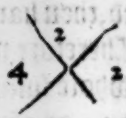


Then examine I the quotient, whiche was 301, where I finde but onelye 4, that do I set at the other side of the crosse, thus.



Then doe I multiple 4 by 2, & it maketh 8, to that doe I adde the remayner

of the diuision (which was 228, and maketh 12) and they two make 20, wherein I finde twice 9, and 2 remayning, that 2 must I set at the head of the crosse, thus.



Then doe I examine the whole number to bee diuided, which was 136280, where I finde twice 9, and 2 remayning, which I set at ẏ foote of the crosse, thus.

And bicause that it doth a=gree with the figure at the head of the crosse, I knowe ẏ the diuision was well wrought.

☞ Mayster. This is the common proofe: how be it, the more certaine working is by the contrary kinde, as to proue Diuision by Multiplication, thus.

A proofe by Multiplication.

Multiplie the quotient by the diuisor, and if the summe that amounteth bee equall to the summe that shoulde bee diuided, then haue you well diuided, else not.

Howbeit, this must you marke, that if there remayned any thing after the diuision, that must you adde to the summe that amounteth of the multiplication: as in our first example the quotient was 13, and the diuisor was 28: Nowe

Nowe multiplie the one by the other, and the summe will bee 364 : to that if you adde the one that remained after the diuision, then will it bee 365, whiche was the summe that shoulde be diuided, and therefore I knowe that I haue well done.

Scholer. Nowe will I proue the same in the second example, whose diuisor was 452, and the quotient 301 : these doe I multiply togither, and there amounteth 136052, to whiche if I adde the 228 that remayned, then will it bee 136280, whiche was the whole summe to bee diuided, and therefore I perceiue that I haue well done.

Mayster. This is the surest waye to examine Diuision by Multiplication : and contrariwise the surest proofe of Multiplication, is Diuision.

And therefore nowe will I shew howe you may proue Multiplication by Diuision.

When you haue ended Multiplication, and would knowe whither you haue well done or not, set the grosse summe that amounteth of the Multiplication ouermost, and diuide it by the multiplier : and if the quotient be the same number that should bee multiplyed, then haue you well wrought, else not : as in that exam- A proofe of Multiplication by Diuision.

ple where we multiplied 264, by 29, the groſſe ſumme was 7656.

Now if you will know whether that multiplication be true, you ſhall diuide that 7656, by the Multiplier, 29: and you ſhall perceiue that the quotient will bee 264, and that is a token that you haue well wrought.

Scholer. By your pacience I will proue that: and firſt I ſet downe the groſſe ſumme & the Multiplier, not after the rule of Multiplication, but after the rule of Diuiſion, for nowe that number is become the diuiſor, that was before the multiplier, I ſhall ſet them therefore thus.

7656
29

Then ſhall I ſeeke how many times 2 in 7, that may bee 3 times, and 1 remayneth: but then may not 9 bee founde ſo often in 16, therefore muſt I take a leſſer quotient, yͭ is to ſay, 2. then ſay I twiſe 2 maketh 4, whiche I take out of 7, and there remayneth 3. then doe I cancell 7 and 2, and ouer 7 I write 3, and in the quotient I ſet 2, ſo yͤ figures ſtande thus.

3
~~7~~656 (2
~~2~~9

Then ſay I forth, 2 times 9 make 18, whiche I bate out of 36, and there reſteth 18, then cancell I 3, and ouer

ouer him set 1, and likewise I cancell 6 and 9, and ouer them I set 8, so that thus stande the figures.

```
   1
  3 8
 3 6 5 6 ( 2
  2 9
```

Then we I set forwarde the diuisor by one place, and seeke a newe quotient, that is to say, how many times 2 are in 18, which I finde to be 9 times, but then cā I not finde 9 so many times in 5, therefore I take a lesser quotient, as to say 8: but yet is ẏ to greate, for if I take 8 times 2, out of 18, there remaineth but 2, and I can not finde 8 times 9 in 25, therefore yet I take a lesser quotient, that is 7, which is also to greate, for if I take 7 times 2 out of 18, there resteth 4, but nowe I can not take 7 times 9 out of 45, therefore yet I seeke a lesser quotient, as to say, 6: then say I, 6 times 2, make 12, that I take out of 18, and there remaineth 6, so I cancell the 18 & the 2, and write 6 ouer 8, thus.

```
  1 6
  3 8
 3 6 5 6 ( 26
  2 9 9
    2
```

```
 1 6
 3 1 2
 3 6 5 6 ( 26
 2 9 9
 2
```

Then say I forth: 6 tymes 9, maketh 54, that take I out of 65, and there remaineth 11, and the figures stande thus.

Then muste I set forth the

diuisor agayne, and seeke a newe quotient, which will bee 4 : for though I maye finde 2 in 11 fiue tymes, and 1 remayne, yet I can not finde 9 so often in 16, therefore I set the figures thus.

1
1 8
3 6 1
7 6 5 6 (264
2 9 9 9
2 2

And the 4 in the quotient I multiply into the figures of the diuisour, saying : 4 times 2, maketh 8, which I take out of 11, and there resteth 3, therefore I cancell the 11 and the 2, and set 3 ouer the first place of 11, thus.

1
1 8 3
3 6 1
7 6 5 6 (264
2 9 9 9
2 2

And then doe I say forthe, 4 tymes 9 maketh 36, which I take from 36, and there remayneth nothing, so that the quotient of this diuision, where 7656 is diuided by 29, is 264, which doth declare, that if 264 bee multiplyed by 29, the summe will be 7656. And thus I perceyue nowe how bothe Multiplication is proued by Diuision, and Diuision also by Multiplication.

Maister. Now haue I ended the fiue moste cōmon kindes of Arithmetike : for as touching Mediation, Duplation, Triplation, and such other, they are no seuerall kindes of Arithmetike,

tike, but are contayned vnder the other: for Mediation is contayned vnder Diuiſion, and is nothing elſe but diuiding by 2: and ſo are Duplation and Triplation contained vnder Multiplication: for Duplation is nothing elſe but multiplying by 2, and Triplation is multiplying by 3, of which I will only propoſe examples, for ẏ rules you haue hearde already.

If you woulde mediate or diuide by 2, this ſumme, 4531010, you ſhall ſet 2 for the diuiſor, & worke as you learned before, as thus. An example of Mediation.

4 5 3 1 0 1 0 (
2

Then I finde 2 in 4 two times, therefore my quotient muſt bee 2: ſo I cancell 4 and 2, and remoue the diuiſor forwarde, thus.

~~4~~ 5 3 1 0 1 0 (2
~~2~~ 2

Then agayne I finde 2 in 5 twice, and 1 remayning, ſo I write 2 againe for my ſecond number of the quotient, and cancell 5 and 2, and ouer 5 I ſet 1, thus.

1
~~4~~ ~~5~~ 3 1 0 1 0 (2 2
~~2~~ ~~2~~

Then remoue I the diuiſor forwarde and ſeeke a newe quotient, which is 6: then ſay I 6 tymes 2, make 12, take ẏ out of 13, & there reſteth 1, ſo I cancell 2

and 13, and ouer 3 I
set 1, thus. 1 1

Then remoue I the 4 5 3 1 0 1 0 (226
diuisor forwarde, and 2 2 2
seeke a new quotient
which is 5, then take I twice 5 out of 11, and
there resteth 1, so I
cancell the 2 and the 1 1
laste figure of 1, and 4 5 3 1 0 1 0 (2265
let ẏ first stand thus. 2 2 2 2

Then remoue I the
diuisor forwarde, and seeke a newe quotient,
which is 5: then take I 2 fiue tymes out of
10, and there resteth nothing.

Then remoue I
againe the diuisor 1 1
forwarde, thus. 4 5 3 1 0 1 0 (22655

But bicause I 2 2 2 2 2
can not finde the di-
uisor in the number ouer it, I muste set a ci-
pher in the quo-
tient, and re- 1 1
moue the diui- 3 5 3 1 0 1 0 (226550
sor to the nexte 2 2 2 2 2 2 2
place, as appea-
reth in the figure before.

Then seeke I a new quotient, which I finde

to

to be 5, for so many times may I haue 2 in 10. Then haue I fully ended this Mediation or diuision by 2, and the quotient is this 2265505, which is the halfe of 4531010, as you may trie by Duplation: for double that quotient, or multiplie it by 2, and the same number will amount.

Duplation.

I will no longer tarry about these, seeing they are but members of the other kindes. But here now will I teach you certaine easie formes both of Multiplication and of Diuision, and first of Multiplication.

Easie formes of multiplication.

If you would therfore multiply any summe by 10, you shall neede to doe no more but adde a cipher before his first place: as for example: 36 multiplied by 10, make 360.

Likewise if you will multiplie any summe by 100, put two ciphers at his beginning.

So if you would multiply any summe by a thousande, adde three ciphers to the beginning of it.

Scholer. This doe I well perceyue, and also the reason of it.

Mayster. I will omit all reasons till our next meeting, when I shall tell you the reason of all other partes of Arithmetike also: and as to our matter nowe, looke (as I haue tolde you)

you) that you both remember it, and also often practise it.

But if you would multiplie any number by 5, marke first whether the nūber be euē or odde: and if it be euen, take the halfe of it, and write a cipher at the beginning of it, as for example: I woulde multiplie 2 5 6 4 by 5, I take ẏ halfe of it, whiche is 1 2 8 2 (as you may knowe by Mediation) and before it I set a cipher, thus, 1 2 8 2 0, and this is 5 times 2 5 6 4.

And thus may you doe with any other euen summe, that you would multiply by 5.

But if the summe be odde, as for example 2 5 6 3, then must you take the lesser halfe of it, or (if you will) take away 1 from the first fygure, (as here take 1 from 3) and then take the halfe of the reste, and sette before it 5: as of 2 5 6 3, the lesser halfe is 1 2 8 1, for heere I take but 1 for the halfe of 3: and if I putte 5 before that lesser halfe, then haue I multiplyed it 5 times, as thus, 1 2 8 1 5.

Sc. What meane you by the lesser halfe?

Mayster. There is no iust halfe of any odde number, therefore if we diuide an odde number into two partes as nighe equall as canne bee, yet will the one halfe exceede the other halfe by one, as for example. The two moste neerest

halfes

halfes of 9 are 5 and 4: and likewise of 15, are 7 and 8, where you see, that the one parte still is greater than the other by one. Nowe it is easie to know which is the greater halfe, and which is the lesser halfe.

Scholer. Then I perceyue you, and can doe likewayes (I doubt not) with any summe. For if it be not very easie to parte into halfe, then will I do it by Mediation easily ynoughe.

Mayster. That is a sure way. And nowe haue you learned howe to multiplie easilye by 5, 10, 100, 1000: and of like manner may you doe with any other of that sort.

But nowe if you will multiplie by 20, 30, 40, and so forth: or by 200, 300, and suche like, where there is one cipher in the first place, or many orderly in the first places, you shall take awaye those ciphers, and multiplie the summe onely by the other figure or figures, (if they bee many) and then at the beginning of the summe that amounteth, shall you set so many ciphers as you tooke away.

Example of 2873, which I woulde multiplie by 300. First I cast away the two cyphers from the multiplyer, & I multiply the summe by onely 3 that is lefte, and it amounteth to 8619: before which I put ẏ two ciphers that

I tooke away before, and then is it 861900. And that is the summe that amounteth, when 2873 is multiplied by 300.

Scholer. And if there were two or more figures beside the cyphers, I must onely take away the cyphers, and multiply by the other figures, as I learned before: as if I would multiply 93648 by 25000, I should take away the three cyphers, and multiply the same by 25, & then at the beginning of that totall summe, should I adde the three ciphers againe.

Mayster. Euen so: but and if it chaunce the number that should be multiplied, or bothe the summes, as well the number that shoulde bee multiplyed as the multiplier, to haue cyphers in their first places, euermore caste away the cyphers, and worke by the rest. But remember to restore as many cyphers to the amounting summe, as you bated before, as in this example: 30200 shal be multiplied by 206: I shall onely take away the two ciphers from the greater number, and then multiplye 302 by 206, and afterwarde adde the two cyphers againe. But if I woulde multiplie the same 30200 by 2060, I shall not onely take away the two ciphers from the number that shoulde bee multiplyed, but also I may take

awaye

away the one cipher from the multiplyer, and then muste I adde 3 ciphers to the summe that amounteth: but take heede that you take away no cipher that commeth after anye signifying figure, as in this laste example, you may not take awaye that in the fourthe place of the higher number, neyther that in the thirde place of the multiplyer: howe bee it, yet this you may doe: If one cipher or more come in the middest of your summes, you may multiply by the other figures, and ouerskippe them, but so, that you giue euerye figure his due place, as thus: I will multiplie 3026 by 2004, therefore I set them thus.

$$\begin{array}{r} 3\,0\,2\,6 \\ 2\,0\,0\,4 \\ \hline 2\,4 \\ 1\,2\,0\,8 \end{array}$$

And thus I multiplye them: firste 4 times 6 make 24. I set the 4 vnder ye first place, and keepe the 2 in my minde, or write it downe for easie remembraunce: then say I againe: 4 times 2 maketh 8, and 4 times 0 maketh 0, then 4 times 3 make 12.

But nowe when I come to the next cipher, bicause that it multiplyeth nothing, I let it go, and likewaies the seconde cipher, but then when I doe come to the 2, and multiply it into the 6 of the ouer number, you muste take heede

(according

(accoꝛding as I taughte you in Multipli‑cation) that the firſt number amounting of the Multiplication, bee ſet vnder the multiplyer right, and the other oꝛderly towarde the lefte hand accoꝛding as you may ſee in this example.

3 0 2 6
2 0 0 4
2 4
1 2 0 8
1 2
6 0 4

Where if you had expreſſed the ciphers after the common rate, then ſhoulde the figures ſtande as followeth.

But in effecte all is one, ſaue that the firſte way by ouerſkipping of the ciphers, is the ſhoꝛter and eaſier way: foꝛ, that, in effecte they bee bothe one, the addition of the parcelles will declare, which in bothe will appeare thus.

3 0 2 6
2 0 0 4
2 4
1 2 0 8
0 0 0 0
0 0 0 0
1 2
6 0 4
6 0 6 4 1 0 4

And nowe will I make an ende of this matter.

Sch. Sir, I thanke you: foꝛ I ſee great eaſe in this wayes of Multiplication, and if you canne ſhewe mee ſuche like in Diuiſion, you ſhall greatelye further mee.

Mayſter.

Mayster. Yes, I will teache you some easie wayes in Diuision also, and firste this: If you woulde diuide any summe by 10, you shall onely with your penne make a square line, betweene the firste figure of your summe and the seconde, and then haue you done: for the whole number that followeth the lyne, standeth for the quotient, and the figure that is before the line, is the remayner: as for example, 3648 diuided by 10, will stande thus. 364|8 Where 364 is the quotient, and betokeneth that so manye tymes are 10 in 3648: and the 8 after the lyne, is the remayner, which can not bee diuided into 10, but by breaking it into fractions, wherewith I will not meddle yet.

Easie formes of Diuision.

And so likewayes if you would diuide any summe by 100, with your penne, you shall cut away the two first figures: and if you woulde diuide by 1000, you muste cut away the 3 firste figures: & so of any other diuisor, whose laste figure is 1, and the other be ciphers, looke how many ciphers the diuisor hathe, and so manye figures at the beginning shall you cut awaye with the squire lyne, and they stande alwayes for the remaner, bicause they are lesse than the

diuisor, and can not bee diuided by it, and the other figures that be behinde the line, stande for the Quotient.

But now if your diuisor haue any other figure in his laste place than 1, and in all his other places haue ciphers, looke howe many ciphers there bee, cut away so many of the firste figures of the number that shoulde bee diuided, and diuide the reste that followeth the line, by that figure that is in the last place, as if it were the whole diuisor.

Example of 64284, which I woulde diuide by 300, here muste I cut away the two firste figures, (for so many ciphers my diuisor hath) and muste diuide the reste by 3, which is the figure in the laste place of the diuisor. Firste therefore I part away the two first figures, & ÿ summe standeth thus. 642|84

Then doe I diuide 642, by 3, and the quotient is 214, for in 6 I finde twice 3, and in 4 once, and 1 remayning, which 1 with the 2 next before, doth make 12, wherein I finde 3 foure times: and this is a ready way to turne shillings into poundes: for sithe one pounde doth containe 20 shillings, I muste diuide the whole number of shillings by 20, therfore easily to doe it, I see that my diuisor hath

one

on e cipher, and therefore I cut away one figure from the beginning of the whole summe of shillings, and then doe I mediate or diuide by 2, the other figures or summe that followeth.

Scholer. I will put an example.

If you woulde diuide 64287 shillings by 20, that is to say: if I woulde turne so many shillinges into poundes, I muste cut awaye the first figure, that is 7, and diuide the reste, that is 6428 by 2, so shall the Quotient bee 3214, whereby I knowe that 64287 shillings, make 3214 poundes, and 7 shillings remayning.

Mayster. Nowe proue by Multiplication whether you haue well done or no.

Scholer. The quotient is 3214, whiche I doe multiplie by the diuisour 2, and it doth amounte to 6428.

Mayster. Heereby maye you perceyue not onely that you haue well done, but also howe by diuision you may turne shillinges easilye into poundes: and contrarie wayes, by Multiplication you may turne poundes into shillings.

But heere shall you see amongst diuers mē, diuers formes of suche diuision, but if you marke what I haue tolde you, you shall per-

tayne easily all their wayes: for some men do not cut away so many of the first figures of the sum ẏ they would diuide, as there are cyphers in the first places of their diuisor, but they set al their ciphers orderly vnder ẏ first places of the number ẏ they woulde diuide, and then wt the other figure (or figures if they bee many) they diuide the resse of their summe. Example. If they woulde diuide 725931, by 3400, they set their summes thus.

In other maner of the abridgement.

72543	1	
34	00	

And then do they diuide orderly till they come to the ciphers: for there they staye and ende their worke, as in this example: They seeke how often 3 may be founde in 7, whiche is 2 times, and one remayning, therefore they set 2 in the quotient, and cancell 3 and 7, & ouer 7 they set that 1 ẏ remayned, thus.

1		
$\cancel{7}$25931		(2
$\cancel{3}$4	00	

Then, do they go forth saying: 2 times 4 maketh 8, whiche they take out of 12, and there remayneth 4, thus.

$\cancel{1}$4		
$\cancel{7}\cancel{2}$5931		(2
$\cancel{3}\cancel{4}$	00	

Then renue they the diuisor forwarde, and seeke how often 3 may bee founde in 4, which

is but once, and 1 remayneth, then set they 1 in the quotient, and cancell 3 and 4, and ouer them they set that 1, thus.

1
~~1~~ ~~4~~
~~1~~ ~~2~~ 5 9 3 1 (21
~~3~~ ~~4~~ 4 0 0
~~3~~

Then take they once 4 out of 15, and there resteth 11. Or else more easily: Take once 4 out of 5, and there resteth 1, so they cancell the 4 and 5, and set 1 ouer them, thus.

1
~~1~~ ~~4~~ 1
~~1~~ ~~2~~ ~~5~~ 9 3 1 (21
~~3~~ ~~4~~ ~~4~~ 0 0
~~3~~

Then set they forth the diuisor agayne, and seeke howe manye tymes 3 are in 11, which they finde 3 tymes, and 2 remayning: so they set 3 in the quotient, and cancell 11 and 3, and ouer them setteth 2, thus.

~~1~~ 2
~~1~~ ~~4~~ ~~1~~
~~1~~ ~~2~~ ~~5~~ 9 3 1 (213
~~3~~ ~~4~~ ~~4~~ 4 0 0
~~3~~ 3

Then do they multiplie 4 by 3, which maketh 12, that with draw they out of 29, & there resteth 17, of which ye 7 must be set ouer the 9, and the 1 ouer the 2, thus.

1
~~1~~ ~~2~~
~~1~~ ~~4~~ ~~1~~ 7
~~1~~ ~~2~~ ~~5~~ ~~9~~ 3 1 (213
~~3~~ ~~4~~ ~~4~~ ~~4~~ 0 0
~~3~~ ~~3~~

And now are the two ciphers next ensuing, so that the diuisor can no more be set forward, and therefore is the diuision ended, and the remayner is 1731.

Nowe the quotient, whiche is 213, doth declare, that if you diuide 725931, by 5400, you shall finde it therein 213 times, and there remayneth 1731, so shall you finde it, if you worke as I taught you, by cutting away the two first figures, bycause of the two ciphers. But this must you marke (as you maye perceyue by this last example) that if there bee left any other remainer in the summe that was behinde the squire line, that the remayner muste bee set to the latter ende of the first remayner whiche was cut awaye with the squire line: as if you would diuide 725931 by 3400, after the forme that I taught you, thẽ would your summes appear thus.

```
     1
    1 2
  1 4 1 7
  7 2 5 9 | 3 1 (2 1 3
  3 4 4 4
    3 3
```

So that 17, whiche remayneth after the line, must be set to the 31 (that was cut away with the line) in higher places, as you see heere: where that 17 with the 31, do make 1731.

And heere would I make an ende of Diui-

sion

sion, sauing that there commeth to my minde one late inuention of easie Diuision, which I will briefly set forthe to you, so that if you finde ease in it, you may vse it. Bicause that the hardest poynt in Diuision, is the ready and easie finding of the quotient number: and againe, if that be truly knowen, all the rest is but lyght to bee done: therefore this wayes shall you quickly and truely finde the quotient.

An other inuention of easie Diuision.

First write the nine figures of number: I meane 123456789, not along as I haue set them now, but vp and downe as in this forme. And at the left side of them drawe a long lyne, as you see here: Then consider the diuisor, by which you intende to worke, and set it on the lefte side of the longe line, right againste 1, and for a distinction drawe a lyne vnder it: then multiplye your diuisor orderly by eche of those figures, beginning with 1, and so go downeward till you haue ended all. And looke what doth amount of the multiplication of eche figure into the diuisor, then write it againste the figure whereby you did multiply.

| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |

Scholer. By example I may perceyue it

 better,

better.

Mayster. Take this example 263845 to bee diuided by 64, then must I sette the 9 figures as I sayde before, and the diuisor must I set against the 1, thus.

64	1
	2
	3
	4
	5
	6
	7
	8
	9

Thē must I multiply yͤ diuisor by eche figure orderly: firste by 2, and it maketh 128, which I must set against 2 at yͤ left hand.

Then multiply 68 by 3, and it maketh 192, which is sette agaynste 3. Then 4 tymes 64, make 256, that set I by 4. Thē say I, 5 times 64, make 320, that set I against 5. Thē 6 times 64, make 384, that sette I against 6. Then 7 times 64, make 448, which I set against 7.

Further I say: 8 tymes 64, make 512, which I set by 8. And laste of all I say: 9 times 64, make 576, which I sette against 9. And then they will stande thus.

64	1
128	2
192	3
256	4
320	5
384	6
448	7
512	8
576	9

And so is the table ended, by which you may easily finde the Quotient, as you shall see by example nowe.

Doe you set downe yͤ numbers (as you lear-

ned

ned before) according to the order of Diuision.

Scholer. That is thus. 263845

Mayster. Now looke 64

what number standeth ouer the diuisor, reckening thereto all them that bee behinde it towarde the left hande.

Schol. Then are there ouer the diuisor 263.

Mayster. That is iuste: now seeke in the table on ẏ left side, whether you can finde 263.

Scholer. It is not there.

Ma. Then take ẏ number ẏ is next to it, beneth it: I meane a lesser number than 263, but of all the lesser numbers that the table hathe, take you that that goeth nighest to 263.

Scholer. That is 256.

Mayster. So is it: and marke this euermore, when you can not finde iustlye in the table that summe that is ouer your diuisor, then note that that is next benethe it in any summe that is in the table, and looke at the right hande of the line what figure or digit that is against ẏ summe, & take that digit for your quotient, & then worke on, as you learned before: for now haue I tolde you the whole vse of this table.

Howbeit, yet that you may bee sure to vnderstande it: I will see you ende this example of Diuision by it.

Now therefore beginne againe.

Scholer. Firste I set downe the summes after the common manner, thus.

2 6 3 8 4 5
6 4

Then doe I looke ouer the diuisor, and finde there 263. Nowe to knowe howe many times 64 may bee taken out of 263, I resorte to the table aforesaide, and seeke for the number 263, but it is not there, therefore as you badde mee, I take a lesser number, the nexte to it that I can finde in the table, and that is 256, which number hathe againste it on the right hande this digitte 4, which I muste take for the firste figure of my quotient.

Then doe I (as I learned before) multiplye that quotient into euery figure of the diuisor orderly, withdrawing the summe thereof, amounting out of the ouer summe: as here I say first: 4 times 6 make 24: so I take that out of 26, saying: 4 out of 6, remaineth 2, which I write ouer the 6: then 2 out of 2 remaineth nothing, then cancell I 2 and 6, and also 6 in the diuisor, and the summe standeth thus.

2
~~2~~ ~~6~~ 3 8 4 5 (4
~~6~~ 4

Then doe I likewise say forthe: 4 times 4 make 16, which I take out

of

of 23, and there resteth 7 to be set ouer 3, & that 3 with the 2 behinde it and & the 4 vnder it, muste also bee cancelled, as you see heere.

$$\begin{array}{l} \quad\;\; \cancel{2}\,7 \\ \cancel{1}\,\cancel{2}\,\cancel{6}\,\cancel{3}\,8\,4\,5\;(4 \\ \quad\;\; \cancel{6}\,\cancel{4} \end{array}$$

Then haue I done with the figure of the quotient.

Mayster. Nowe set forwarde your diuisour, and seeke a new quotiente, as you sought this.

$$\begin{array}{l} \quad\;\; \cancel{2}\,7 \\ \cancel{2}\,\cancel{6}\,\cancel{3}\,8\,4\,5\;(4 \\ \;\; \cancel{6}\,\cancel{4}\,4 \\ \quad\;\; 6 \end{array}$$

Scholer. Thē thus standeth the figures, so that ouer the diuisour I set 78, whiche I seeke in the table, and can not finde it: therefore I take the nexte lesser, and that is 64 the diuisor it selfe.

Mayster. So muste you doe when there is none other.

Sc. Then agaynst it I finde this digit 1, whiche I muste set in the quotient before 4, thus.

$$\begin{array}{l} \quad\;\; \cancel{2}\,7 \\ \cancel{2}\,\cancel{6}\,\cancel{3}\,8\,4\,5\;(41 \\ \;\; \cancel{6}\,\cancel{4}\,4 \\ \quad\;\; 6 \end{array}$$

Then multiplie I 6 by 1, and it is but 6 still.

Mayster. You neede not go about to multiplie Note.

tiplye when the quotient is 1, for 1 doth neither multiply nor diuide, but in such case onely subtract the diuisor out of the number ẏ is ouer it.

Scholer. Then I take 4 out of 8, and there resteth 4, and 6 out of 7 there remayneth 1, so I cancell those numbers, & write the remayners ouer their places, thus.

```
      1
    x 7 4
  x 8 3 8 4 5 (41
    8 4 4
      8
```

Then set I forwarde the diuisor againe, thus.

```
  1   .
  x 7 4
x 8 3 8 4 5 (41
  8 4 4 4
    8 6
```

Where I see ouer ẏ diuisour 144, whiche I seeke in the table, & find it not: therefore I take ẏ number in ẏ table ẏ is next thereto, beneath it, whiche I finde to be 128, agaynst whiche in the right side I finde 2, which I take for my quotient, & that doe I multiplie firste into 6, and thereof commeth 12, which I take out of 14, and then remayneth 2, that 2 I sette ouer 4, and cancell the other figures, 1, 4, and 6, thus.

```
      x 2
    x 7 4
  x 8 3 8 4 5 (412
    8 4 4 4
      8 8
```

Then say I forth: 2

times

times 4 are 8, whiche I take out of 24, and there remayneth 16, of which I write the 6 ouer 4, & the 1 ouer 2, and cancell 2,4, and 4, thus:

```
      1
     1 2
   2 1 4 6
2 6 3 8 4 5 (4 1 2
   6 4 4 4
     6 6
```

Nowe againe I set forward yͤ diuisor thus. And seing ouer it 165, I seeke that in yͤ table, but finde it not, therefore I take yͤ next lesser, which is 128, against which I find 2: that do I set into the quotient, and by it I multiply first 6, & thereof cōmeth 12, which I take out of 16 & there resteth 4, then cancell I 1,6, & 6, and ouer 6 I set 4, thus.

```
      1
     1 2
   2 1 4 6
2 6 3 8 4 5 (4 1 2
   6 4 4 4 4
     6 6 6
```

```
      1
     1 2 4
   2 1 4 6
2 6 3 8 4 5 (4 1 2 2
   6 4 4 4 4
     6 6 6
```

Then doe I multiply 4 by 2, and it maketh 8, which I take out of 45, and there remayneth 37, as in this example

```
        1 3
      2 2 4
   2 1 4 6 7
2 6 3 8 4 5 (4 1 2 2
   6 4 4 4 4
     6 6 6
```

And nowe haue I done.

Mayster. Well, nowe I see that you can worke by this kinde of diuision, as farre forth as I taught you.

Scholer. Yea sir, I thanke you, and I finde in it much ease and certainnesse.

Mayster. Yet one thing I doubt whether your perceyue: what if you did finde in the table the number that standeth ouer the diuisor, what would you next doe?

Scholer. I thinke I shoulde take a digit agaynst it on the left hand for the quotient.

Mayster. So is it: and as often as you seeke in the table and finde your number iuste, the digit agaynst it is your true and iust quotient. I call that a true quotient also, if it bee the right quotient that you should take, though your diuisor multiplyed by the same, doe not clearely subtract the number ouer it, but there dothe somewhat remaine, as it chaunced in all the examples that you did worke by. But if it should chaunce (as it doth often) that your diuisor multiplied by your quotient, doe subtract cleane the number ouer it, then call I that quotient not onely a true quotient, but also a iust quotient, bicause it doth iustlye consume the number ouer the diuisor: and that chaunceth euermore when the number ouer the diuisor is iustly

Marke the diuersitie betwene a true quotient and a iust quotient.

iustly founde in the table.

Scholer. This I shall remember.

Maister. But yet one easie pointe more I will tell you in this sorte of Diuision, therefore marke it well.

When you haue founde in the table, other the same summe that is ouer the diuisor, other ẏ next beneath, (for lacke of ẏ other) then looke what digit standeth againste it, take that for your quotient. And bicause it is some paine to multiply the diuisor by the quotient, you shall not neede to doe it, but onely take the number that you founde in the table, and subtract that from the ouer number: for if you doe multiply the diuisor by the quotient, that will be the nūber that shall amounte, therefore is this waye more easier.

Scholer. So is it, and also more certaine for such as I am, that might quickly erre in multiplying, especiallye being smally practised therein.

Mayster. Then proue in some briefe example whether you can doe it, and so will we make an ende.

Scholer. I would diuide 38468 by 24, therefore firste I set ẏ table as here followeth.

$$\begin{array}{r} 38468 \\ 24 \end{array}$$

Then

190

24	1
48	2
72	3
96	4
120	5
144	6
168	7
192	8
216	9

Then set I the two summes of Diuision, thus. And ouer the diuisor I finde 38, which I seeke in the table and finde it not, therefore take I the next beneth it, which the table hath, and that is 24, the diuisor it selfe: against which is set 1, which I take for the quotient, which I set in his place. And now I neede not to multiplye the diuisor by it, but onely to withdraw the diuisor oute of the 38, that is ouer it, and so remaineth 14, as thus.

```
 14
~~3~~~~8~~468 (1
~~2~~4
```

Then set I forwarde ỹ diuisour, and finde ouer it 144, as appeareth: then seeke I that number in the table and finde it, & against it is 6, therefore I set 6 before 1 for my quotient, and I take that 144 for the iuste multiplication of the diuisor by that quotient, and therefore without any newe multiplication I doe subtract ỹ 144, from the other 144, & there resteth nothing, as you may see,

```
 14
~~3~~~~8~~468 (1
~~2~~~~4~~4
  2
```

```
 ~~1~~~~4~~
~~3~~~~8~~~~4~~68 (16
~~2~~~~4~~~~4~~
 ~~2~~
```

Therefore

Therefore I set forward the diuisor: but se=ing it will not bee in the next place, (for then ouer 2 would be nothing) I set it forward twice, as you see here.

```
  1 4
3 8 4 6 8 ( 16
2 4 4
  2 2
```

And for bicause that I could not set it in the next place following, therefore I set a cipher in the quotient, as you see.

Then looke I ouer the diuisor, and finde 68, which I can not finde in the table, therefore take I the next beneth it, which I finde in the table, and that is 48, and againste it standeth 2; which I take for the quotient. And then without anye multiplying of the quotient in=to the diuisor, I doe sub=tract that 48 from 68, and there resteth 20, as heere appeareth.

```
1 4   2 0
3 8 4 6 8 ( 1602
2 4 4 4 4
  2 2 2
```

And so haue I ended the whole diuision.

Mayster. In very greate summes to be di=uided by greate diuisors, I thinke there is no better way thā this for any man to vse, though hee bee neuer so expert. And that especially, if one great diuisor be often to be occupied about diuiding many and diuers greate summes. As

commonly happeneth in Aſtronomicall workings, and Geometricall, about the ſignes, both ſtraight and reuerſed: as if it be your fortune and deſire to wade to the profoundneſſe of Geometricall and Aſtronomicall calculations demonſtratiue, you will ſoone confeſſe. Whereof, an other tyme ſhall better ſerue to ſpeake. Now can you ſufficiently ſkill in theſe kindes of Arithmetike. And now for the farder vſe of theſe two laſte, that is Multiplication and Diuiſion, I will briefly ſhew you the feate of Reduction by the way.

REDVCTION.

REduction is, by which all ſummes of groſſe denomination may bee turned into ſummes of more ſubtile denomination: And contrary wayes, all ſummes of ſubtile denomination, maye bee brought into ſummes of groſſer denomination.

Scholer. What call you groſſe denomination, and ſubtile denomination?

Groſſe denomination

Mayſter. That I call a groſſe denomination,

tion, whiche dothe conteyne vnder it many o-ther subtiler or smaller: As a pounde in re-spect to shillings, is a grosse denomination: for it is greater than shilling, and conteyneth ma-ny of them. And shillings in comparison to poundes, are a subtile denomination, for bicause they are lesser than poundes, and many of them are conteyned in one of the other: as so, likewayes of other things, what so euer thing is compared to other, if it be greater and conteyneth manye of them, it is a grosser de-nomination: but if it be lesser, so that many of them are in the other, then are they called subtile denominations: whereby you may per-ceyue, that one denomination may bee called a grosse denomination, and also a Subtile, (that is to say, a great and small) in diuerse comparisons. For shillings cõpared to pounds are a Subtile or small denominatiõ: but com-pared to pennies, they are a grosse or great de-nomination. Subtile de-nomination.

Scholer. Now I vnderstande the name, I pray you teach me the vse.

Mayster. The vse is easily learned, if you remember what you haue learned before. For if you will reduce anye summe of a grosse de-nomination, into a summe of a smaller or sub- To reduce grosse de.

omination o subtile. tiler denomination, you muste consider howe many of that Subtiler denomination do make one of the Grosser denomination, and by that number or numerator doe you multiplie the other summe: as if you would reduce 20 pounds into shillings, you muste consider that in a pound are included 20 shillings, therefore multiplie the one 20 by the other 20, and there will amount 400, whereby you may knowe, that in 20 pounde are conteyned 400 shillinges. Likewayes if you woulde reduce 30 shillinges into pennies, considering that in 1 shilling, are 12 pennies, you muste multiplie 30 by 12, and it will bee 360: whereby you finde, that in 30 shillings, are conteyned 360 pennies. And thus may you reduce any grosse denomination into a more subtiler, by multiplication, if you know howe many of the lesser doe make the greater: of whiche thing I will anon giue you a briefe table for the most accustomed kindes of money, waights, measures, and time, and such like, whereby you may knowe howe often eche Subtile denomination is conteyned in the Grosser, when you shall neede it for the foresaide kinde of Reduction. And also the same shall serue you, if you woulde reduce any summe of a Subtiler denomination, into a summe

To reduce Subtile

a summe of a grosser denomination: For in such reduction you must consider (as in the other forme) how many of the smaller doe make the greater, and by that number muste you diuide the other summe, and the quotient will declare how many of the greater denominations are comprehended in that summe: as for example: if you would know how many shillings are conteyned in 3240 pens, consider that 12 pennies doe make 1 shilling: you muste diuide that 3240 by 12, and your quotient will be 270, whereby you knowe that so many shillings are in 3240 pennies. But and you would knowe farther, how many poundes are in those 270 shillings, seeing that euery pounde conteyneth 20 shillings, diuide that 270 by 20, and it will be 13, and 10 remayning, whereby you may knowe, that in 3240 pennies, or 270 shillings, are 13 poundes & 10 shillings. For euermore the remaynder must be named by the name or denomination of the summe that was diuided, whiche in this place were shillings. And thus may you do with any other kindes of denominations.

Denomination too grosse.

Wherefore to the intente that you maye haue a light knowledge in the commō coynes, weightes, measures, and such other, I haue

prepared heere a briefe table, whiche shall suffise to you at this tyme, till hereafter at more conuenient oportunitie I maye instructe you more exactly in the same.

Note (gentle reader) these values of English coines as they were whe this Author first published this book, but in our time, (namely Anno 1570) they are much diuerse. Therefore something to pleasure thee in this purpose, I haue caused at the ende of the booke, a table to bee annexed of our coynes, and their values currant in our time, within this realm of Englande.

A table for English coynes. Anno. 1540.

Englishe coines.

A Soueraygne.	A quarter Noble.
Halfe a Soueraygne.	A Croune.
A Royall.	Halfe a Croune.
Halfe a Royall.	A croune.
A quarter Royall.	A Grote.
And olde Noble.	A harpe Grote.
Halfe an olde Noble.	A penny of 2 pens.
An Angell.	A dandy pratte.
Halfe an Angell.	A penny.
A George Noble.	An halfe pennie.
Halfe a George Noble.	A Farthing.

The

The valew of Englishe coynes.

A Soueraygne is the greatest english coyne, and contayneth 2 Royalles, or 3 Angelles, eyther 9 halfe Crounes, or 4 Crounes and an halfe, that is to say, 22 ß. 6 d. | The value of English coynes.

Halfe a Royall is equall with a Royall.

A Royall containeth an Angell and a halfe, that is to say: 11 ß, 3 d.

Halfe a Royall contayneth 5 ß, 7, d, ob.

A quarter of a Royall contayneth 2 shillings, 9 d. ob. q.

An olde Noble called an Henrye, is worthe 2 Crownes, or a Noble and a halfe, that is 10 ß.

Halfe an olde Noble is worth 5 ß.

An Angell containeth a Crowne and halfe, or 3 halfe Crownes, that is 7 ß, 6 d.

Halfe an Angell is worth 3 ß, 9 d.

A Noble, called a George, is worth 6 ß, 8 d.

Halfe a Noble is worth 3 ß, 4 d.

A quarter of a Noble (which in ye olde Statutes is called a farthing) contayneth 20 d.

A Crowne containeth 5 ß: & the halfe Crowne 2 ß, 6 d. How bee it there is an other Crowne of 4 ß, 6 d, which is knowen by the rose side: for the rose hath no Crowne ouer it, as in the other Crowne, but it is enuironed on the 4 quarters with 4 floure deluce, whereby you

may best know it. But I will returne to speake of the value of the coynes, for I intende not now to describe the formes of them. Nowe of golde are there no more common coynes?

In siluer the greatest is a Grote, which containeth 4 pennies. Then is there an other Grote called a Harpe, which goeth for 3 d. Then next is a Penny of 2 d. And thē a Dandiprat, worth 3 halfepens. Next it a Penny, then halfe a penny, and laste and leaste of all a Farthing, whose coyne is on the one side a crosse, and on the other side a purculles. This I tell you, bicause I see many that can not know a Farthing from a small halfe penny.

Nowe haue I tolde you all the Englishe coynes bothe of golde & siluer, but yet of ẏ three moste cōmon valowers of mony spake I nothing: that is to saye, of Poundes, Markes, and shillings, which though they haue no coynes, yet is there no name more in vse than they: of which the shillings containeth 12 pennyes or 3 grotes: and the pounde 2 olde Nobles, 3 George Nobles, or 4 Crownes, that is to say, 20 ß, A marke, two George Nobles, that is 13 ß. 4 d.

Here woulde I now expresse the valewes of sundry other coynes of diuers countryes, but

for

for thre causes I nowe refrayne. The firste and chiefest is, bicause they are not currant by the Statutes of this Realme. An other cause is, by reason they are so uncertaine, that they be neuer long at one rate. And againe they are so different in so manye places, that it were matter ynough for a greate boke, to speake sufficiently of them all. Howe bee it, yet bicause you shall not bee altogither ignorant of them, I will shew you the valewes of some that are moste in use: and firste of Fraunce.

The moste common money are Deniers, Soulx and Frankes, 12 Deniers make 1 ß. 20 soulx make 1 Frank, so ỷ as you see, these 3 kindes are like in the rate to pennies, shillings, and poundes with us, but that this is the difference, that their Denier is but the 9 parte of our penny, and so their soulx (cōmonly called sowses,) go 9 to our shilling, and 9 of their Frankes to an Englishe pounde of mony: So that 3 of their Frankes make a noble. And by those 3 may you practise, how to reduce French mony into Englishe mony. And as for the rest of their coynes I will omit till an other time, when I intende to shew you the rate of sundry other kindes of mony. Frenche coynes.

But nowe as for the coynes of Flaunders

Flaunders coynes.

be so changeable, that you muste knowe them frō time to time, els you cā not reduce them into our mony, certainely. But yet bicause ye you shall haue an example of their monye to exercise you withall, you shall take those yt bee most common, as Stiuers bothe single and double, Grotes Flemmishe, Carolus, and Gyldens: A Flēmishe Grote is a litle aboue 3 farthings Englishe. A single Styuer is 1 d, ob, q̄. The double Styuer is 3 d, q̄. The siluer Carolus single, 2 d, q̄, q, c. The double siluer Carolus is 4 d, ob, q̄, q. Thē is there also ye Carolus Gylden, which is worth 20 Styuers. And the Flemmishe Noble is worth 3 Carolus Gyldens, and xij. Styuers.

But I will let them passe nowe, exhorting you to practise to reduce those kindes into English mony, according as I haue set forth here following: 2160 deniers, make 240 d: or 20 ß. 3240 deniers, make 360 d, or 30 ß: 8352 deniers, make 928 d, or 3 li, 17, ß, 4 d: 2160 soulx, make 240 shillings: and so of other in like rate.

But if you will reduce Flemmishe monye iustly, you muste reduce it first into the smallest parte of Englishe mony that is in ye coyne, as for example. If I woulde reduce 368 double

styuers

styuers into English money, considering that a double Styuer conteyneth 3 d. q, you shal first loke howe manye q, bee in a double Styuer, and you shall finde them 13, therefore multiply the summe of the styuers by 13, and then haue you their value in farthings, whiche is 4784. Nowe if you diuide that by 4, then will there appeare the number of pens: but better it were to diuide it by 48 (for so many farthings are in 1 shilling) and then will the quotient declare the summe of the shillings.

Likewise if you woulde reduce any summe of single styuers into English mony, you must multiplie the summe first by 13, and then haue you a certaine summe, whiche summe if you diuide by 8, then will amounte the summe of pennies: or if you diuide it by 96, the summe of shillings will appeare,

But this marke in all diuision, when ye doe reduce to bring one denomination into an other, if there bee any remayner after the diuision, that must bee named by the denomination of the grosse summe that was diuided: as for example: I woulde bring 254 q into pens, therefore I do diuide that 254 q by 4, for so many farthings make 1 pẽny, & the quotient is 63, which is the summe of the pens, and then remayneth

mayneth yet 2, whiche are farthings still, as one may proue by diuiding. And this must bee marked in all Diuision, namely whē it is done for Reduction.

VVeights. Thus muche haue I sayde of Money, now will I shewe you in like sorte the distinction of waightes, after the statutes of Englande, where the least portion of waight is commonly a Grayne, meaning a grayne of corne or wheate, drie, and gathered out of the middle of the eare. Of these graynes in times passed, 32 wayed iust 1 pennie of Troy, and then was but 20 pennies in an Ounce. But nowe are there 46 pennies in an Ounce, so that there are not fully 14 graynes in one pennie.

A graine. A pennie of Troy. An Ounce.

But nowe of Ounces after Troye rate (whiche is the standarde of Englande) 12 doe make 1 pounde.

Haberdepoise vveights. But commonlye there is vsed an other waight called Haberdepoise, in whiche 16 ounces make a pounde. Therfore when you would reduce ounces into poundes, you must consider whether your waightes be Troy waightes or Haberdepoise: and if it be Troy waight, you must diuide your ounces by 12, to bring them to poundes: but if it bee Haberdepoise, you muste diuide them by 16. Nowe againe, there

bee

be greater waights which are called an Hun- dred, halfe a hundred, and a quarterne, and also halfe a quarterne. &c. A hundred vveight.

Scholer. Why? so there may bee reckened 30 pounde, 40 pounde, 200 pounde, and such innumerable.

Mayster. All these are numbers of waight, but they haue not common waightes made to their rate, as the other haue. And againe, these that I did name, are not iuste in number as they seeme by their name, for an hūdred is not iust 100, but is 112 pounde. And so the halfe hundred is 56: the quarter 28: and the halfe quarter 14. And this is the common waightes vsed in most things that are solde by waight.

Howbeeit there are in some things other names: as in wolle, 28 pounde is not called a quarterne, but a Todde: and the 14 pounde is not named halfe quarterne, but a Stone: and the 7 pounde, halfe a stone. Other names, bicause they differ in many places, and agree in fewe, I let them passe. VVooll vveightes. Todde. Stone.

But a Sacke of wolle by ẏ Statutes is limitted to be 26 Stone. Sacke.

Now in Cheese, though it bee solde by the hundred, and by the stone in some places, yet the verye weightes of it are Cloues, and Cheese vveights.

Weyes

Cloue. VVey. Weyes: so that a Cloue shoulde contayne 7 pounde: and a Wey 32 Cloues, that is 224 poundes. Howbeit some Statute bookes saye, that a Cloue should be 6 pound: and some say also, that a Weye dothe conteyne 36 Cloues, and that is commonlye vsed for the common Wey, for the common Wey is of 256 ℔. that is 36 cloues, reckening 7 ℔. to the cloue, and there is 4 ℔ ouer waight. Let this suffice you at this time, touching waightes.

Measures for liquor. Now of waightes are made other measures, both for graine and lyquor. For a pounde in waight maketh a pynte in measure, so that 8 pounde (or 8 pyntes) doe make a Gallon: halfe a Gallon is named a Pottell: & halfe a Pottell is called a Quarte, whiche conteyneth two pintes. Nowe aboue a Gallon the next measure is a Fyrkin: then a Tertian, a Kilderkin or halfe Barrell, and a Barrell. And by those measures are solde commonly Ale, Bere, Wine, and Oyle, Butter, and Sope: Salmon, Herings, and Eeles.

A pinte. Gallon. Pottell. Quart. Fyrkin. Tertian. Kilderkin. Barrell.

Ale measures. But as these be vnlike things, so the measure of their vessels do differ: for the measures of Ale are as followeth.

Of Ale	the Fyrken	conteineth	8	gallons.
	ye Kilderkē		16	
	the Barrel		32	

Of

Of Beer.	the Fyrkin	cōtai-neth	9	Galōs.
	ye Kilderkin		18	
	the Barrell		36	

Sope measures, bothe Fyrkin, Kilderkin, and Barrell, should bee all equall to Ale measures. Moreouer the Statutes doe limitte the waight of euery of those three vessels, beyng emptye, (Sope measures.)

1 Barrell	to	26	poundes.
halfe Barrell	weyghe	13	
1 Fyrkin	emptye,	6 $\frac{1}{2}$	

Hearings also be solde by the same measures that Ale and Sope bee solde by. (Hearings.)

Hearings also are solde by the tale, 120 to the hundred, ten thousand to the last.

Salmon & Eeles haue a greater measure. (Salmon and Eeles.)

Salmō & Eeles	the butte	hol-deth	84	gallons.
	ye barrell		42	
	halfe bar.		21	
	the firkin		10 $\frac{1}{2}$	

How be it, some Statutes did limitte Eele vessels equall with Hearing vessels.

Now as for wine vessels seldome are smaller than Hogges headdes, which are of 63 gallons: euery Hogges hed is two Barrels: yet there are many other wine vessels, but of them all, see this table, and marke the measures one (VVine measures.)

to

to an other.

Of wine and oyle				
the Rondelet	holdeth	18 ½	Gallons.
the Barrell		31 ½	
ye hogges hed		63	
the Tertian		84	
the Pype		126	
the Tonne		252	

Tertian. But you shall marke, that there bee other kindes of Tertians: for there be Tertians (yt is to saye) Thirdles of Pypes, of Hogges headdes, and Barrels, as well of other things as of wyne.

A Butte. Also of Malueseyes and Secke, &c. the halfe tonne is not called a Pype, but rather a Butte.

And thus much haue I thought meete to tell you at this tyme.

Scholer. And is this alwayes true?

Mayster. I haue tolde you how it shoulde bee, but howe it is I may not say: howe they doe differ daily from their iust measure, ye Gagiers can tell you better than I. But I will let this passe now, and speake briefly of the other measures.

And as of weyghtes there did spring the liquide measures, (whereof I spake laste) so of the same springeth drye measures: as Peckes, Bushels, Quarters, and such like, whereby

Dry measures.

are

are measured corne and like graines: also salt, lime, coales, and other like: And this is the order and quantitie of them.

A Pecke is the measure of two Gallons. A pecke.

A Bushell containeth foure Peckes. Bushell.

A Quarter holdeth eight Bushells. Quarter.

A Wey conteyneth six quarters. VVaye.

These are the common names and measures, but in diuers places there bee diuers sortes.

The bushell in many places is 2 bushels: but then is the bushell there called a Stryke. And in some places halfe a quarter is called a Cornoke. But these diuersities are to many to tell you briefly them all. And againe, sithe they are againste the law and Statutes, I counte them vnmeete to bee vsed. Strike. Cornoke.

But nowe remaineth yet an other kinde of Measure, whereby men meate length & bredth, and thicknes, and those are an Ynche, a Foote, and such other: whose names and quantities this table sheweth. Measures to meate lengthe, breadth, & thickenes.

3 Graines of barly in lẽgth, make an inche. An ynche.

12 Ynches make a foote. Foote.

3 Foote make a Yarde. Yarde.

3 Foote and 9 Ynches make an Elle. Elle.

5 Yardes and a halfe, make a Perche. Perche.

1 Perche in bredth, and 40 in length, doe

 make

make a Rod of lande, which some call a rode, some a yarde lande, and some a Farthendele.

Acre.
{ 2 Farthendels, make halfe an acre of grounde.
{ 4 Farthendels make an Acre.

Here mought I tell you many things els touching measure: and also howe to reduce straunge measures to our measures, but bicause it can not well bee done without ẏ knowledge of Fractions, which as yet you haue not learned: I will let them passe till an other tyme, when I shall instruct you in Geometry, wherin I should be enforced els to repeate the same often agayne.

The partes of tyme.

Scholer. But yet sir of the partes of time, I pray you tell me some what.

A Daye. An houre. VVeeke. Moneth. yeare.

Mayster. You knowe that a naturall day hath 24 houres, and euery houre hath 60 minutes. It needeth not to tell you that 7 dayes make a weeke, and 4 weekes make a common moneth, and 13 monethes make a yeare, lacking 1 daye and certaine houres and minutes. But of that I shall instruct you hereafter.

Here will I make an ende of Reduction for this time, which though it be counted no kinde seueral of Arithmetike, yet you see it is no lesse needefull to bee knowen, nor easier to be done, than of any of the other.

Scholer. Mary sir, it seemeth vnto mee much harder than any other sort, for it requireth the knowledge of so many things: but nowe sir when you see time, I am readye to learne forth, for as much of Reduction as you haue taught mee, I remember, but and if I doe at any time forget, I shall haue recourse to the tables, which you haue set forth for me.

Mayster. So doe you, for it will not bee remembred without exercise. Now Progression will I begin.

PROGRESSION.

Although vntill this daye the moste parte of wryters haue defyned Progression as a compendious kinde of Addition, yet truely it is not so: for Progression (as the verie nature of the worde doth informe any man) is a going forwarde and proceeding in numbers, and that regularly and orderly, whose place is aptly chosen to bee verie neare, or rather next after the exposition of the foure principall partes of Arithmetike, for in it after a moste easie maner, are all the

foure former parts exercised and practised: and not onely Addition, as customablye is done. Whiche custome hath beene the cause, why it hath so specially beene named a kinde of Addition, and defined to be a quicke and briefe Addition of diuerse summes, proceeding by some certaine and reasonable order.

You shall also vnderstande, that there are infinite kindes of progressions, but for you (as yet) two are sufficient to bee exercised in: of whiche the one I call Arithmeticall, and the other Geometricall.

Arithmeticall Progression.

Arithmeticall progression is a rehearsing or placing downe of many numbers, number after number, in such sorte, that betweene euery two next numbers rehearsed or placed downe, ẏ differēce, diuersity, or excesse, be equall & alike.

Scholer. Sir, I thanke you for that you haue both opened vnto mee what Progression is, truly, and also why it is heere placed. But I pray you wt an example make plaine youre definition.

Mayster. Examples cannot want, seeing all reasonable creatures naturally vse the order of one kinde of Arithmeticall progression, (whiche therefore is also named Naturall) when so euer they distinctly doe count or num-

ber

ber any multitude one by one, ſaying: 1.2.3.4. 5.6. whereby the proceeding from number to number, and euerie one ſurmounting and exceeding his fellowe next before by a like quantitie (whiche heere is 1) declareth the ſame to be Arithmeticall progreſſion. And for the more plainneſſe, I ſet it downe in this maner.

The common exceſſe. 1 1 1 1 1

The Progreſſion. 1 2 3 4 5 6

Scholer. This is moſte euidente. And I thinke that I am able to tell you nowe of any progreſſion Arithmeticall propounded, what is that common exceſſe or difference whereby it proceedeth, if this order be kept in it.

Mayſter. What ſay you of 3.6.9.12.15. ?

Scholer. They exceede eche other by 3. And that maye I ſet downe in ſuche euident order, as you did your example of Naturall progreſſion, in this wiſe.

The common exceſſe. 3 3 3 3

The Progreſſion. 3 6 9 12 15

Mayſter. And doe you not alſo nowe perceyue, that the whole table of Multiplication may bee made by the order of progreſſion A-

rithmeticall? either if ye will begin at the first number of any of them on the lefte hande, and so proceede right ouerthwart: or at any of the first numbers of the vpper rowe, and goe directly downewarde?

Scholer. I praye you let mee consider the thing a little, and I will answere you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	10	40	50	60	70	80	90	100

By this triall I perceyue it now very well: for the common excesse or difference betweene any two next, is continually as much as the first number of euery rowe, either from the left hande ouerthwart taken, or from anye of the vppermost ouerthwarte rowes downwarde.

Mayster. Nowe then if of any such progression

ression you woulde speedely knowe the totall summe, much quicklyer than by common additions rules: first tell howe many numbers there are (which numbers here wee call places or parcels) & if they be odde, write their summe downe by it selfe, as in this example, 2, 4, 6, 8, 10, 12, 14, where the numbers are 7, as you may see, therefore set downe 7 in a place alone: thē adde togither the first number and the laste, as in this example: adde 2 to 14, and that maketh 16, take halfe of it, and multiply by the 7, which you noted for the number of the places, and the summe that amounteth, is the sūme of all those figures added togither; as in this example: 8 multiplyed by 7, make 56: and that is the summe of all the figures.

To know the totall summe of an Arithmetiiall progressions.

Scholer. That will I worke by an other example. I would know how much this sūme is 5, 8, 11, 14, 17, 20, 23, 26, 29, I tell the places and they are 9, that I note. Then I put the first number 5, and the laste 29, togither, and they make 34, I take the halfe of it, that is, 17, and multiply by 9, and it maketh 153. That you say is the summe of all the numbers.

Mayster. So shall you finde it if you trie it.

Scholer. How shall I trie it?

Mayster. By your common addition: for

 if you

if you adde all the parcells togither, you shall se the same summe amounte, if you did worke well. And that manner of addition trieth all kindes of summing any Progression.

Sc. Then can I summe a progression, if the numbers of the partes bee odde. But what if they bee euen? as in this example, 1, 2, 3, 4, 5, 6, 7, 8?

Mayster. When the number of the parcells is euen, then note that also as you did before, and likewaies adde the firste summe to the laste, and by the halfe of the number of the places do you multiply it: as in your example, the parcells are 8, that note I: then adding the first summe to the laste, there amounteth 9, that do I multiplye by the halfe of parcells, that is by 4, and it maketh 36, which is the summe of the 8 parcells.

☞ But if you will take one rule for these both, do thus. Multiply the halfe of the one by the other whole, and the summe will amounte all one. For sometime it chaunceth that the number of the parcells bee odde, so that their halfe can not bee taken: and sometime it chaunceth the Addition of the firste number and the laste, to bring forth an odde number, so that the halfe of it can not bee taken: but they will neuer bee

bothe

bothe odde.

Scholer. Then I perceyue this, if there bee no more longing to it.

Mayster. As accustomably it hath bene taught, this hath ben the chiefe and onely exercise in Progression vsed. But that you maye perceyue how diuers waies and to how greate profit so simple a thing (as this Arithmeticall progression is) may be considered and vsed, I will here propounde you sixe propositions, of which foure of thẽ were inuented by a friende of mine, and neuer before this published: and the firste two, were neuer to my knowledge written of, but by three men.

Scholer. This dothe greatelye encourage me to be attentife vnto your wordes, seeing I shall not onely bee instructed at your handes in the common knowen rule of this excellente arte, but besides that, so aboundantly in other new rules informed, as my very entrance shall seeme to passe a greate many mens farther studie, and longer continuaunce. Therefore sir, I beseeche you, let mee knowe your sixe propositions.

Mayster. These they are.

To know the last number without proceding by

continuall addition, tyll you come vnto it, so that the common excesse, the first number and the number of the places be knowen.

2 The first number of the progression and the laste being knowen, with the common exces, to finde the number of the places.

3 The exces being giuen, and the first or last, to know the quantity of any middle number, whose place is giuen from the first or laste.

4 The totall summe beyng gyuen, and the firste and laste, to finde out the number of the places.

5 The totall summe of any Arithmeticall progression being giuen, and the first and laste, to finde out the common excesse.

6 The totall summe being giuen, and the mutuall exces, with the number of the places, to gyue the first or laste number of the same progression.

Many mo considerations could I propounde you in these Arithmeticall progressions, but these are sufficiēt to giue you occasiō to thinke, that rules of knowledge & artes are infinitelye capable of enlargement.

Scholer. Happy were I, if I did but well vnderstande that which is all readye inuented & writen. And yet in my simple fantasie these things offer them selues (in manner) to bee studied for about progression, therefore I pray you to proceede to the rules answering to these propo-

propoſitions.

Mayſter. I will orderly for euery of theſe ſix propoſitions giue you rules, and with euery one an example, vnleſſe the plainneſſe and eaſineſſe neede no farther exemplifying.

For the Solution of the firſt. Multiplie the exceſſe by a number leſſe by 1 then the number of the places, and the ofcome adde to the firſt number, ſo ſhall you haue the laſt number, whiche is ſought for. » »

As (for example,) if there were ſeuen places in a progreſſion Arithmeticall, whoſe continuall increaſe or mutuall exceſſe were 4, and the firſt number were 5, and I woulde know what the laſt and ſeuenth number is: I multiplie 6, which is 1 leſſe than 7, (the number of the places) by 4, thereof commeth 24, which I adde to 5, that maketh 29: and that is the laſt number, whiche I deſired to know. And this you may ſtraight way proue, by continuall proceeding from 5 till the ſeuenth place, encreaſing euery one by 4, as thus.

5 9 13 17 21 25 29.

Lo heere, the laſt, beeing alſo the ſeuenth is 29.

Scholer. I perceyue all ready one good propertie in this rule, whiche in all workes is to be

deſired

desired: yͭ is, it will ease one from great labour, if a progression were propounded of a hundred or two hundred places or mo. And also it is very easie to worke, & most necessary for the totall summe finding, in a verie long progression.

2 Mayster. The second rule is this. From yͤ last subtract the first, the remayner diuide by the common excesse, to the Quotient adde 1, & you haue the number of the places, whiche you woulde knowe: as in this progression.

6 11 16 21 26 31

If I knowe onely 6 and 31, and that they encrease by 5, then according to the rule, from 31 I subtract 6, there remayneth 25: which 25 I diuide by 5 (the common excesse) the quotient commeth forth 5, to which I adde 1, that maketh 6: and so many are the places, as you see.

Scholer. This rule is so easie, that I were much to blame, if I coulde not remember it.

3 Ma. The thirde proposition may alwayes thus be soluted: Multiply yͤ excesse by a nūber lesse by 1, than the distance of yͤ place is from yͤ first or the last number giuen: the ofcome adde to the first, if the distance bee reckened from the first, & the first also knowne or subtract from the last: if the distance be from the last counted, and the last giuen also, & that which cōmeth forth,

either

either in that Addition to ẏ first, or subtraction from the last, is the number sought. As for example, I propound you this progression.

8	15	22	29	36	43	50	57.

And for the apt considering the maner of this question, I will note ouer euery place his distance from the first: and vnder euerie place his distance inclusiuely from the last, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if ẏ excesse wherby this progression stādeth, bee knowne to be 7, & the first number giuen, being 8, I would know what nūber standeth vnder 4, ẏ is to say in the fourth place. I multiply 7 by 3 (whiche is lesse by 1 than the number of the place propounded) that yeldeth 21, to which I adde 8 (the first number) so commeth 29: which I say to belong to the fourth place, as ye see in the example it also doth: or if in the thirde place from the last, you woulde knowe what number in this example shoulde stande, the last number beeing knowne to bee 57, and the common excesse 7, then by 2 (which is lesse by 1 than the place propounded) I multiply 7, ẏ giueth 14: which I subtract from 57, so remayneth 43: which appertaineth to ẏ third place

place inclusiuely reckened from the last, and so my example giueth you.

Scholer. I perceyue right good vse of this rule: for if I had forgotten what the first number were, and remember still but the last, the common excesse, and the number of the places, then might I come by the knowledge of my first number againe.

And me thinketh, that it differeth not much from the first proposition, sauing that whiche you make here a middle number, there was made the laste: and also in this poynt it differeth, that in it the last was onely sought, and no consideration had in numbring the places from the last, as here I marke in your numbers noted vnder your progression.

Mayster. And thinke you not, the middle numbers of a progression standing of a hundred or three hundred places or moe, maye as much cumber a man to come to the knowledge of them by continuall encreasing from the first (by the common excesse) or abating from the last continuallye (the common excesse) as the verie small numbers in a shorter Progression woulde doe?

Scholer. Yes sir, that I thinke right well, and therefore I am glad of this newe framed

proposion

proposition, and the manner of the working of it.

Mayster. The rule of the fourth is this: Adde the first and the laste togither, and by the ,,
ofcome diuide the totall summe. Double the ,,
Quotient, and that will bee the number of the ,,
places. ,,

Scholer. Then if in a Progression, whose summe were 207, and the firste number 12, and the laste 57, if I adde 57 and 12 togither, that maketh 69: and by it I diuide 207, the Quotient will be 3, which I double, and so I haue 6, and so many must bee the number of the places that this progression standeth on.

Maister. Whether it be so or no, how will you trie?

Scholer. Halfe 6, which is 3, being multiplyed by 69, muste make 207, the totall summe, if 6 bee the number of the places. For so the whole worke of youre rule in summing any Arithmeticall progression did enforme me. I will then multiply 69, by 3, thus.

69
3
207

It commeth forth iustly.

Maister. I muste much herein commende your promptnes, bothe in memory and in well applying youre rule: although in manifest

wordes

wordes it did containe no such matter.

Scholer. Sir, I pray you heare mee frame one example more.

Mayster. I am well pleased, so that ye bee short, for you make mee more longer here, than willingly I woulde haue bene: but I can not perceyue how I could haue omitted any thing as yet, without your greate lacke thereof.

Scholer. If I had receyued 85 poundes of certaine men, but of howe many I haue forgotten, yet I remember that the firste gaue me 7 ℔, and the laste 27 ℔, and euery payment after other did rise by a like summe. And the man for whome I receyued this mony, conditioned with me, ỹ of euery payment I shoulde haue twelue pens for my labour: nowe vnlesse I can by arte finde the truthe of this case, I am like to lose the moste part of my reward.

Mayster. I perceyue you can handsomly frame an example, which shoulde concerne your owne gaine: I pray you let mee see how you would doe iustice in this poynt.

Scholer. I adde the first 1

and the laste togither, that ~~2~~7 17

maketh 34: by which I di- ~~8~~ ~~5~~ (2 ———

uide 85, thus. ~~3~~ ~~4~~ 34

Why how now? Sir, here 8

is

is a remnāt of 17, in which 34 can not be had: so that nowe I am in the bryers for doubling of my quotient, and farewell then bothe my Justice and a good lumpe of my gaines.

Mayster. Yee are neuer the farther from the matter, though it fall into a Fraction. For you shall vnderstande, that the fraction which of any such worke proceedeth, is euer halfe of one such, as the vnits of the Quotient before are. And that you may trie, if you double that which so remaineth, for then it will bee equall to your diuisor, as if ye double 17 (the remnant) it maketh 34, and your diuisor also was 34, this noteth the remainder to bee halfe of one.

Scholer. Nowe I am glad of this harde example. For with it I haue a generall rule for the Fraction that may happe in this worke. So that the quotient being two and a halfe, I duble that, and it maketh 5, therefore shoulde my gaine bee 5 shillinges. And to bee sure, (by your leaue) I will trie it, for I will multiplie halfe of 34, (which is the firste and laste number ioyned togither) by 5, thus. It is most true (I see) that I should leese nothing by the

17
5
85

fozmer wozking.

Mayſter. The fifth pzopoztion hath this
" rule appertaining vnto it: By the fourth rule
" finde the number of the places, that being done,
" from the laſte ſubtract the firſte, and the reſi-
" due diuide by a number leſſe by 1, than the
" number of the places, and the quotient will
ſhew the exces which is ſought foz.

An example hereof ſhall be this: If yee had diſburſed 685 poundes to a certaine number of men, you neyther can tell howe many they were, oz howe much the ones mony exceeded his next befoze, but you are ſure that the exces was equall betweene euery two next: and alſo you remember that yͤ firſte had 19, and the laſt 118 poundes, how would you finde both the number of the men and the exces, continually obſerued in the ſucceſſion of their paiments.

Scholer. Your rule doth plainely bid, firſt to finde the number of the places, which I will doe accozding to the fourthe rule. I adde 19 and 118 togither, thus.

	118
	19
	137

By this 137, I diuide 685, thus.

	~~1~~ ~~5~~
	~~6~~ ~~8~~ ~~5~~ (5
	~~1~~ ~~3~~ ~~7~~

Seeing there is no fraction, but a whole nũber, being 5, I

double

double that, and then muste the number of ŷ places be 10. Now from the last I subtract the first, as 19 from 118, thus: And so remayneth 99.

$$\begin{array}{r} 118 \\ 19 \\ \hline 99 \end{array}$$

This 99 I diuide by a number lesse by 1 than the number of the places, and seeing the places were 10, I diuide 99 by 9, thus.

$$\begin{array}{l} \cancel{9}9 \\ \cancel{9}9 \end{array} (11$$

ŷ quotient is 11, & so was the excesse, if I haue followed your rule right.

Mayster. You haue wrought euery part of this question both well in order, and truely in the practise of your rules.

Scholer. I will then set it downe also formably, so that the number of the places, the excesse and the totall summe maye streight appeare, as your first example stode.

11	11	11	11	11	11	11	11	11	11	The common exces.
19	30	41	52	63	74	85	96	107	118	The progression.

That the places bee 10, and that from the first to the last the common excesse is 11, I perceiue moste euidentlye, but whether the totall summe be 685, I haue not yet proued, whiche I will nowe doe. I adde 19 and 118 togither, that maketh 137: I multiplye that by

 halfe

halfe the number of the places, thus.

	137
	5
	685

All things agree moste exactlye, so that I am perfect ynough in these rules, if I forgette them not againe.

6 Mayster. Use maketh all things perfect.

,, Your sixt rule is this. By the uumber of
,, the places diuide the totall summe, double the
,, quotient, and that will bee the first and the last
,, ioyned in one summe. Then by a number lesse
,, by 1 than the number of the places, multiplye
,, the excesse, that ofcome subtracte from the firste
,, doubled quotient, and the halfe of the residue
,, is the firste number. The laste number you
,, maye diuersly finde out, as by the first of our
,, sixe rules, or by subtracting this firste number
,, from the summe whiche heere contayned bothe
,, the firste and the laste ioyntly, (or thirdly) by
,, continually adding the excesse.

Scholer. I pray you make this somewhat more plaine with an example.

Mayster. If euerye moneth in the yeare (counting them nowe as thirtene) you gayned cleerely 40 shillings more than you did the moneth next going before, and at the yeres ende you finde the whole gaine 5720 shillings, but

but yee remember not howe muche either the gaine of the first moneth or the laste was, by this rule it may bee tryed out.

Scholer. So that here yee seeme to applie the 13 moneth to thirtene places, the 40 shillings euery one more than the other nexte before it, to bee the common excesse, and 5720 ß. to bee the totall summe.

Mayster. It is true: by 13 then I diuide 5720, in this maner.

~~5~~
~~5~~ ~~7~~
~~5~~ ~~7~~ ~~2~~ 0 (440
~~1~~ ~~3~~ ~~3~~ 3
~~1~~ 1

I double this quotient, so haue I 880 for the first, and the laste summe ioyned togither by 12, which is lesse by one than the number of the places, I multiply 40, (the common excesse) so commeth 480.

40
12
———
80
40
———
480

This 480 I subtract from 880, so remayneth 400: halfe whereof is the firste Number whiche we desired to knowe: that is 200.

And as for the laste number I can giue you it three wayes: As by the first of my sixe rules. I multiplie the excesse by a number lesse by 1 than the number of the places: as 40 by 12, that giueth 480, whiche I adde to the firste

beeing 200, so shall the last be 680.

The same summe commeth forthe, if yee subtract 200 from 880.

And thirdly, if I beginne at 200, and so procede, encreasing by 40, I shall at the thirtenth place haue 680, as thus.

200. 240. 280. 320. 360. 400.
440. 480. 520. 560. 600. 640.
680.

Scholer. I thanke you moste hartilye for these sixe rules. Nowe if it bee your pleasure I woulde heare and learne somewhat of Progression Geometricall.

Mayster. There are yet very many rules and propositions, which fall into this Arithmeticall progression: but these shall suffice for this time.

And in Geometricall Progression I will be more briefe, both bicause I haue beene so long in this parte of Arithmeticall progression, and also for that it woulde require the knowledge of Rootes, and numbers surde, (wherof ye haue yet learned nothing) if I should frame the like propositions in them as I haue done in these. Therefore I will onely teache you two practises, about it, and so ende the considerations

and

and workes of these progressions. Progression Geometricall is when the numbers increase by a like proportion, that is, if the secōd number containe the firste, 2, 3, or 4 times, and so forth: then the thirde containeth the seconde so many times also: and so the fourth the thirde, and the fifth the fourth: Progression Geometricall.

wherfore I set these three examples.

3,	6,	12,	24,	48,
1,	3,	9,	27,	81,
2,	10,	50,	250.	

Here in the firste example you see, that every number containeth the other (that goeth next before him) 2 times: and in the seconde example 3 times: in the third example 5 tymes. Now if you will know how to finde easilie the summe of any such numbers, doe thus. Consider by what number they be multiplyed, whether by 2, 3, 4, 5, or any other, and by the same number doe you multiplie the laste summe in the Progression.

Scholer. I pray you worke it by this example, 2,8,32,128, 512, 2048, which I haue framed by proceeding from 2, and continuallye multiplying by 4.

Mayster. Then must I multiply the laste summe (which is 2048) by 4 also, and it will bee 8192. Nowe must I ba[.]e from this summe

the first number of the progression, which here is 2, then resteth 8190, which summe I muste diuide by 1 lesse than was the number that I multiplied by. Seing then I multiplyed by 4, I must diuide by 3, so diuiding 8190 by 3, the quotient will bee 2730, which is the summe of all the Progression. And nowe to proue whether you can doe the same, I giue you these numbers to adde by this rule, 3, 15, 75, 375, 1875, 9375, 46875.

Scholer. I can not well tell by what number this Progression doth encrease.

Maister. In anye such doubte, doe thus: ☞ Diuide the second number by the firste, and the quotient will shew you the number that engendreth the Progression.

Scholer. Then is that number in this example 5, for so many times is 3 in 15.

Mayster. So is it. Nowe worke as I taught you.

Scholer. The laste number is 46875, which I multiply by 5, and it yeeldeth 234375, from which I bate the firste number of the Progression, that is 3, and there resteth 234372 which I diuide by 4, for that is one lesse than 5, and the quotient is 58593, which is the whole summe of the progression.

Mayster.

Maister. Nowe that you knowe the summing of Geometricall Progression, I will shew you a cōpendious manner eyther to proceede by, or to finde out the quantity of a number, whose distance from the firste may be very greate, which to doe by continuall multiplication would be very tedious, if the numbers bee greate, and the places many.

An abridgment in progression

Scholer. Nothing can pleasure mee more than breuitie, if it bee plaine.

Maister. I thinke I am not yet in anye poynte so darke or harde, that you neede to feare any obscuritie nowe. The manner is this: set downe of your progression foure or fiue of the firste places, and vnder the firste put a cipher, vnder the seconde 1, vnder the thirde 2, &c. as if ye had a progression encreasing by a fiue folde quantitie: as here, 2, 10, 50, 250, 1250: then vnder 2 I put a cipher, and vnder 10 the figure of 1: vnder 50, 2: vnder 250, 3: vnder 1250, 4: and so forthe if yee will: but to a wise and wary worker, a fewe places were sufficient to proceed by to anye number of places in this sorte, if anye twoo of youre numbers

2	10	50	250	1250
0	1	2	3	4

progressionall bee multiplied the one by the o-
,, ther, and the ofcome diuided by the first of your
,, progression, the quotient is one of your num-
,, bers progressionall, & belonging to that place
,, of your vnder numbers, that is equall to that
,, summe, that is made of Addition togither of
,, your two numbers which stoode vnder these
,, two of your Progressionall numbers, that
,, were multiply-
ed ye one by the other, as in this example

2	10	50	250	1250	6250
0	1	2	3	4	5

If I multiplie 10 by 50, thereof commeth 500, which I diuide by 2, (the firste number of the Progression,) and the quotient is 250: which 250, muste stande in the thirde place, bycause the number whiche standeth vnder 10, is 1, and that vnder 50, is 2: and 2 and 1 maketh 3. Therefore I saye, that 250 belongeth to the thirde place of this progression, as yee see also here it dothe. Moreouer if I multiply

$$\begin{array}{r} 50 \\ 50 \\ \hline 00 \\ 250 \\ \hline 2500 \\ \hline \end{array}$$

50, into it selfe, thereof commeth 2500: that 2500 I diuide by 2, the Quotient is 1250, whiche muste bee sette in the fourthe place: bicause 2 added to himselfe againe, maketh 4, and in our example 1250 occupieth the fourth place.

Scholer. Then for the fifth place, I multiply the Progressionall numbers ouer 2 and 3, one by the other: and for the sixth, I multiplie that ouer 3 in it selfe. &c.

Mayster. Ye must well remember ý these places that we nowe speake of, belong to the vnder numbers, for the true places of the vpper numbers is euer one place more.

Scholer. That I see the reason of, bicause the vnder numbers begin one after, and against the first place of my progression standeth a cypher, so that the 250 whiche you sayde before did belong to the thirde place, I see belongeth to the number of 3 amõg your vnder numbers, but from the true progressions beginning, it is the fourth,

Mayster. You vnderstande mee as I meane. Therefore for your exercise of bothe the rules here giuen for Geometricall progression, I will aske you a question, muche vsed among the common people, (as they haue a

great

great many the like.) If I woulde sell you a Horse, hauing 4 shoes, and in euery shoe sixe nayles, with this condition that you shall pay for the first nayle 1 ob, for the seconde 2 ob, for the thirde 4, and for the fourth 8, and so forth doubling untill the laste nayle. Nowe I demaunde of you, howe much the price of the Horse woulde amount unto?

Scholer. Seeng the Horse hath 4 shoes, and in euerye shoe 6 nayles, I perceyue heere will bee 24 places. If I coulde nowe haue the laste number, I woulde quickly dispatche this question. I will therefore with as fewe multiplications as I can diuise, come to the knowledge of the last number of this Progression. In double I sette foorth then a few of my progression thus.

1	2	4	8	16	32	64
0	1	2	3	4	5	6

If I nowe multiplie the numbers ouer 5 and 6, the one by the other, I shall haue the number of the eleuenth place for the under nũbers, but of the twelft for the upper numbers in whiche my progression standeth, and then that of the eleuenth place under, if I multiplye in it selfe, I shall haue for the 22 place under, but for the 23 of that aboue, whiche I multiply

ply by that ouer 1 of my nether places, and I ſhall haue the 23 of my nether places, and the 24 of the vpper, whiche is the number I ſeeke for.

Mayſter. Mee thinketh you haue forgotten youre rule for abridging your multiplications: for in it, the ofcome euer of any multiplication, is to bee diuided by the firſte of the progreſſion. And you nowe ſpeake of no Diuiſion.

Scholer. Sir I neede not, as my progreſſion beginneth nowe: for if I ſhoulde diuide by 1, it maketh no other quotient, than the nūber is, it doth diuide.

Mayſter. It is very well remembred & noted of you, to your worke then according to your preſcribed maner, which I like well.

64
32
128
192
2048

Sc. I multiply 64 by 32, as here. And it maketh 2048, whiche is the eleuenth place vnder, but the twelfth aboue, and this, I multiply in it ſelfe in this manner. And this is the 22 place vnder, but the 23 aboue. I multiplye this then by 2, as heere.

2048
2048
16384
8192
0000
4096
4194304

And

And this ofcome 8398608, is my foure and twentith place, whiche I haue founde nowe by 3 multiplications.

	4194304
	2
	8388608

Then doe I resorte to the rule of summing this Progression, where I consider that the encrease of this summe proceedeth by multiplication of 2, and therefore I doe multiply the laste summe by 2 also, and it yeeldeth 16777216, from whiche I abate the firste number whiche is 1, and then resteth 16777215, whiche I shoulde diuide by 1 lesse than I did multiply: but seeing that it is 1, I neede not to diuide it, for 1 (as I haue before sayde) doth neither multiply nor diuide, therefore I take that summe 16777215 for the whole summe of the halfe pennies, whiche by Reduction I finde to bee 699050 ß, and 7 d, ob: that is 34952 lb, 10 ß, 7 d, ob.

Mayster. That is well done, but I thinke you will buy no horse at the price.

Scholer. No sir if I bee wise. Yet for my assurance will I take so muche payne, as to come to this last 8388608 by continuall multiplication by 2, as in the page following you maye beholde my worke till I haue done.

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24

done.

Mayster. Well, are yee not almost weary?

Scholer. Well fare my shorte rule, for in trouth it hath more cunning and more ease.

Mayster. Well, then answere me to this question.

A Lorde deliuered to a Bricklayer a certaine number of lodes of brick, whereof he willed him to make 12 walles, of such sorte, that the firste wall shoulde receyue 2 thirdeles of yͤ whole number: and the seconde 2 thirdeles of that yͤ was lefte, & so euery other 2 thirdeles of that yͤ remained: and so did yͤ bricklayer: And when the 12 walles were made, there remaineth one lode of bricke.

Now I aske you, how many lode went to

euery

euery wall, and howe many lode was in the whole?

Scholer. Why sir, it is impossible for mee to tell.

Mayster. Naye, it is very easie, if you marke it well. Marke well that I sayde, that euery wall shoulde receyue 2 thirdels of the summe that was lefte. Nowe take awaye 2 thirdels from any summe, and you must needes graunt that that which remaineth is 1 thirdle of the summe laste before: example of 9, from which if you take 2 thirdels, there will remaine 3, which is one thirdel of 9. Likewaies from 3 bate 2 thirdels, and there will remaine 1.

Scholer This is true, and nowe I perceyue, that the leaste wall had but two lode of bricke.

Mayster. And by the same reason maye you knowe howe many loade euery wall had, according as this figure following doth shew, and likewaies what ẏ whole summe of brickes was: for if you make 12 summes, multiplying by 3, still from ẏ laste remainer, as you see here on the left side of the table, there will appeare all the remainers of euerie wall: and if you multiplie the laste of those 12 summes

by

by 3 also, then will that bee the summe of the loades which were deliuered to the bricklayer.

1	12	2
3	11	6
9	10	18
27	9	54
81	8	162
243	7	486
729	6	1458
2187	5	4374
6561	4	13122
19683	3	39366
59049	2	118098
177147	1	354294

531441

Againe, if you doe double euerye remayner, as you see at the right side of this table, those numbers will shewe the summe of loades that went to eche wall: whereby also you may perceyue, that eche wall was 3 times so greate as the next lesser.

Scholer. Lo, nowe it appeareth easie inough. Now surely I see that Arithmetike is a right excellent arte.

Maister. You will say so when you knowe more of the vse of it: For this is nothing in comparison to other poyntes that maye bee wrought by it.

Scholer. Then I beseeche you sir, cease not to instruct mee farther in this wonderfull cunning.

The Golden rule.

Maister.

The rule of Proportion.

BY the order of the science (as men haue taught it) there should followe nexte the extraction of Rootes of number, which bicause it is somewhat harde for you, yet I will let it passe for a while, and will teache you the feate of the rule of Proportions, which for his excellencie is called the Golden rule. Whose vse is, by three numbers knowen, to finde out an other vnknowen, which you desire to knowe: as thus. If you pay for your borde for three monethes 16 shillings, howe much shall you pay for 8 monethes.

The Golde rule.

Question of bording.

To knowe this and all suche like questions, you shall consider which two of your 3 numbers be of one denomination, and set those two

the

the one ouer the other, so that the vndermost be it y^e the question is asked of: as in my question 3 and 8 bee both of one denomination, for they both bee monethes, & bicause 8 is the number that the question is asked of, I set them one ouer the other, & 8 vttermost, thus, with suche a crooked draught of lines.

3
8

Then do I set the other number whiche is 16, agaynst 3, at the right side of the line, thus.

3 16
8

And nowe to know my question, this muste I doe: I must multiply the lowermost on the left side, by that on the right side, & the summe that amounteth I must diuide by the highest, on the left side. Or in plainer wordes thus: I shall multiply the number of whiche the question is asked (which is called the Thirde number) by the number of an other denomination, (which is called the Seconde) and that summe that amounteth muste I diuide by the summe of like denomination, which is called the First. Then for the knowledge of this question, I multiply 8 into 16, and there amounteth 128, whiche I diuide by 3, and it yeeldeth 42 shillings, and 2 ß remayneth, whiche I turne into pennies, and they bee 24 d, of whiche the thirde part is 8 d, so the third part of 128 ß

The thirde number.
The second number.
The first number.

is

is 42 ß,8 d.: which sum I write at yͤ right hãd of the figure against 8,thus.

3 — 16 ß
8 — 42 ß, 8 d.

Hereby I knowe, that if three monethes boording doe come to 16 ß, that 8. monethes boording will come to 42 ß 8 d.: and likewise of any other like question.

But heere muste you marke, that the first number and the third be of one denomination, and also the seconde and the fourth, for whiche you seeke: or else bee of suche denominations, that you in working may bring them into one. As if a man should aske me this question.

Question of expenses.

Twelue weekes iourneying coste mee 14 Nobles, howe many poundes is that in one yeare? Heere you see no two numbers of one denomination, but yet in working you maye turne them into like denominations, as thus. Turne the one yeare into 52 Weekes, and the fourth summe will bee nobles, by the order of the working. Then to knowe this question, multiplye the thirde summe, 52, by the seconde 14, and the sume will bee 728, that diuide by 12, and it will be 60, and 8 remayning: which if you turne into pennies, they will bee 53 d, and $\frac{1}{3}$ of a penny more: whiche summe of pennies if it bee diuided by 12, will yeeld 4 ß,5 d. and the thirde parte of a pennie: but this 60

Nobles

Nobles (which maketh 20 li) with the 4 ß, 5 d, and q̄, and little more: for the ſumme that anſwereth to the queſtion, and it is the expence of a yeare, and the ſummes will bee thus. Or els, for to note them as they

12 Z 14 Nobles.
52 — 60, 4 ß, 5 d, q̄.

fall oute, preciſelye, with the fraction: and to leaue out the groſſe termes and inartificiall, of a little more, or a little leſſe, thus they muſt bee written. And when you come to the Fractions,

VVeekes.	Nobles.
12	14
52	60 $\frac{8}{12}$

you ſhall by order bee certified, what eyght twelfthes, or two thirds of a Noble is, or what the valuation is of any Fraction, bee it of any coyne, meaſure, waight, or other quantitye, whatſoeuer, whiche hathe vſuall or determined partes knowne.

A generall rule.

And take this for a generall rule, that euermore the Thirde number muſte bee it that the queſtion is ioyned with: and the firſt, the number that is of the ſame denomination, thē muſt the ſecond needes be that number.

Remember alſo, that the place of the Firſt number is the higheſt on the left ſyde: and the place of the ſeconde right agaynſt it on ẏ right

side: the place of the Third number is vnder the first, as by those examples you haue seene.

Scholer. This I trust I can doe.

Mayster. But and if the question be asked thus: In 8 weekes I spende 40 ß, howe long will 105 shillings serue mee? Though the order seeme vnlike, yet take you 105 for the Thirde number, and 40 beeing of the same denomination, for the First, and then 8 for the Seconde. Then multiplie 105 by 8, and it will be 840, whiche if you diuide by 40, it will yelde 21, which is the Fourth number, and sheweth howe many weekes 105 ß will serue, if you spende 40 ß in eight weekes.

The figure of this question is this: as if you should say: If 40 ß serue for 8 weekes, 105 serue for 21 weekes.

Shillings.	VVeekes.
40	8
105	21

Other diuersities there be of working by this rule, but I had leuer that you woulde learne this one well, than at the beginning to trouble your minde with diuerse fourmes of working, sith this way can doe as muche as all the other, and hereafter you shall learne the other more conueniently.

Note. But yet before wee make an ende of this rule

rule,this shall you note, ỷ there is an other or=
der quite contrarie to this that you haue lear=
ned. For in this rule hitherto, euermore looke
how much the Thirde number is greater than
the Firste, so much the fourth number is grea=
ter than the Seconde. And contrarie waies:
looke howe much the Firste summe is greater
than the Thirde, (if it doe chaunce so) so much
is the Second summe greater than the Fourth.
But there is a cõtrary order, as this: That the
greater the thirde summe is aboue the firste, the
lesser the fourth summe is beneth ỷ second: and
this rule you maye call the Backer rule, as in example.

The Backer rule.

Question of bying clothe.

If I haue bought 30 yardes of clothe of 2 yardes breadth, and woulde buy canuas of 3 yardes broade to line it withall, howe many yardes should I neede?

Scholer. Why, there is none so broade.

Mayster. I doe not care for that, I doe put this example onelye for your easie vnderstanding: For if I shoulde put the example in other measures, it woulde bee harder to vnderstande. But now to the matter: If you woulde knowe this question, set your numbers as you did before: but you shall multiplie nowe the first number by the seconde, and that which

ariseth thereof, you shall diuide by the thirde: which thing if you doe here, I meane if you multiplie 30 by 2, it will be 60: which summe if you diuide by 3, there will appeare 20: wherby I knowe, that if 30 yardes of clothe of two yardes broade, shoulde be lined with canuas of 3 yardes broade, 20 yardes of canuas would suffice, as this figure sheweth.

Breadth.	Lengthe.
2	30
3	20

And now bicause ye founde fault at my example, how say you, perceyue you this?

Scholer. Yes sir, I suppose.

An other like question.

Mayster. Then answere me to this question: howe many elles of canuas of elle breadth will serue to line 20 yardes of Saye, of three quarters of a yarde broade?

Scholer. In good faith sir, I can not tell, for I knowe not howe to bring the summes to like denominations.

Mayster. Then I will tell you: sithe there is mention here of quarters, and againe euerie one of the measures bothe elles and yardes may be parted into quarters, we you part them so bothe in the breadth and length, and then put forth the question by quarters.

Scholer. Then I shall saye thus, howe

many

many quarters of canuas of 5 quarters broade, will line 80 quarters of 3 quarters broade?

Mayster. Nowe answere to the question.

Scholer. First I will set the͂ downe in their forme, thus: for 5 is ioyned with the question, and is therefore the thirde number: then is 3 the number of the same denomination, I meane bicause they bee bothe referred to breadth. Nowe I multiply 80 by 3, and it is 240, which I diuide by 5, and it yeldeth 48. Then saye I, that 48 quarters of 5 quarters broade, will suffice to lyne 80 quarters of 3 quarters broade.

Breadth.	Length.
3	80
5	

Mayster. Turne the quarters againe into elles and yardes.

Scholer. Then I saye, that 9 elles and 3 quarters of a yarde of elle broade, will serue to lyne 20 yardes of 3 quarters broade: as this figure sheweth.

Breadth.	Length.
3	80
5	48

Mayster. This rule is so profitable for all estates of men, that for this rule onely (if there were no more but it) all men were bounde highly to esteeme Arithmetike.

By this rule maye a Captaine in warre

worke many things, as I will hereafter instruct you abundantly, onely now I will shew you this one example.

Question of prouision touching an army.

If it shoulde chaunce a Captaine which hath 40000 souldiers, to bee so inclosed with his enemy, that hee coulde haue no freshe purueyaunce of vittailes, and that the vittailes which hee hath, woulde serue that army but onely 3 monethes, how many men shoulde hee dimisse, to make the vittaile to suffise the residue, 8 monethes?

Sc. As you taught mee, I set the numbers thus, saying: If three monethes suffise 40000, to how many will 8 monethes suffise?

Moneths.	Men.
3	40000
8	

Too know this, I multiply the first number 3 into the seconde 40000, and it yeldeth 120000 which summe I diuide by 8, and there will bee, in the quotient 15000, which if I doe subtract from 40000, the remainer will declare that he must dimisse 25000: as this figure sheweth.

Moneths.	Men.
3	40000
8	15000

Mayster. Well, sithe you perceyue nowe the vse of this rule, I will shewe other which

ensue

ensue of the same. And first the Double rule, which is so called, bicause there is in it double working, by which thing onely it differeth from this. The Double rule.

Scholer. Then by an example I shall understand it well inough.

Mayster. So shall you, and let this bee the example: If the cariage of 109 pounde waight 30 miles, doe coste 12 d, howe much will the cariage of 500 weight cost, being caried 100 miles? Of cariage.

S. I pray you shewe me the working of it.

M. You must make 2 workings of it: ye first thus. If a 100 pounde weyght cost 12 d, how much will 500 pounde cost?

Set your figures thus

And multiply 500 by 12, and therof amounteth 6000, which if you diuide by 100, the quotient will be 60, yt is the price of 500 for 300 miles.

VVeight.	Pens.
100	12
500	

Then begin the second worke, saying: if 30 miles cost 60 pens, howe much will 100 miles cost? Set your figure thus.

Then multiply a 100 by 60, wherof amounteth 6000, which beeing diuided by 30 will yelde 200. Then you may saye, that so manye pennies shall

Miles.	Pens.
30	60
160	

shall cost the cariage of 500 pounde waighte 100 miles after the rate of 12 pens for the 100 caried 30 miles.

Question of sowing.

Scholer. Nowe I perceyue it also.

Mayster. Then answere mee to this question, 30 bushels of wheate sowed, yeldeth in one yeare 360, howe many will 80 bushelles yelde in 7 yeare. I meane sowing euery yeare of those seuen, still 80 bushels.

Sc. First I say, that if 30 bushels will yelde 360 in 1 yeare, then 80 bushels will yelde 960 in one yeare. Then for the seconde worke I say: If one yeare yelde 960, then 7 yeare will yelde 6720: as these two figures doe shewe.

Seede.	Encrease.	Yeares.	Encrease.
30	360	1	960
80	960	7	6720

Question of corne.

But nowe sir, if I set forth 30 bushelles of corne to an other man for 7 yeare, agreeing so, that hee shall sowe euerie yeare the whole encrease of the corne, and I at the ende of those seuen yeares to haue the halfe of the whole encrease: I woulde knowe howe many bushels will there amounte to my parte, supposing the encrease to bee after the rate of the laste question, for 30 bushels in one yeare, 360.

Mayster. In such a question you must haue

so

so many seuerall workings, as there be yeares: as for example, in the first yeare 30 bushelles yelde 360: then to know the yelding of the seconde yeare, I must say: if 30 yelde 360, howe many yeldeth 360? Worke by your rule, & you shall finde 4320. Then say for yͤ third yeare: If 30 yelde 360, howe many will 4320 yelde? you shall haue 51840, and so euery yeare, multiplying the whole increase by 360, and diuiding it by 30, the increase of the nexte yeare will amount: as these 7 figures do orderly declare.

a

30—360

b

30 360
360 4320

c

30 360
4320 51840

d

30 360
51840 622080

e

30 360
622080 7464960

f

30 360
7464960 89579520

g

30 360
89579520 1074954240

Where

Where I haue set 7 letters for the 7 yeares, of which the first is set without arte, bicause that is the increase which you doe presuppose: and the last number of eche other, dothe shewe the increase of the yeare that it standeth for, which the letters doth declare: so that the increase of the 7 yeare, is 107495424o bushels: howe many quarters that is, and also howe manye wayes, you may by Reduction soone finde.

Question of movving. Now with one question more I will proue you. If 6 mowers doe mowe 45 acres in 5 dayes, howe manye mowers will mow 300 acres in 6 dayes?

Scholer. If 45 acres doe require 6 mowers, then 300 acres requireth 40. Nowe againe: if 5 dayes require 40 mowers, then 6 dayes needeth but 33 mowers.

Mayster. Why doe you not make mention of the 2 that remayneth in the laste diuision? for the laste part of the question is wrought by the Backer rule, where the first number 5, is multiplied into the second that is 40, whereof amounteth 200, whiche if you diuide by the thirde number 6, the quotient will bee 33 as you sayd, but then will there remayne 2, which can not well bee diuided into 6 partes: howe be it,

be it, you may understande by the sixte parte of 1, the thirde parte of one mans worke, which you must put to the 33, or else you may saye, that 33 workemen will ende all the 300 acres in 6 dayes, saue two mens worke for one daie, or 2 dayes worke for one man. But such broken number called Fractions, you shall hereafter more better perceyue, when I shall wholy instruct you of them.

The rule of Fellowship.

BUt nowe will I shew you of the rule of Fellowshippe or Company, which hath sundry operations, according to the diuers number of the company. This rule is sometime without difference of time, & sometimes there is in it difference of time. Firste I will speake of that without difference of tyme, of which let this be an example. VVithout tyme.

Foure marchantes of one company made a banke of mony diuersly, for the first laide in 30 ℔, the seconde 50 ℔, the thirde 60 ℔, and the fourthe 100 ℔, which stocke they occupied so long, till it was encreased to 500 ℔. Now I demaunde of you, what shoulde eche man receyue at the parting of this mony. Question of a Banke.

Scholer.

Scholer. I perceyue that this rule is like the other, but yet there is a difference, which I perceyue not.

Mayſter. Then will I ſhewe it to you. Firſte by Addition you ſhall bring all the particular ſummes of the Marchantes into one ſumme, which ſhall bee the firſt ſumme in your working by the Golden rule, and the whole ſumme of the gaines by that ſtocke ſhall bee the Second ſumme. Now for yͤ Thirde ſūme, you ſhall ſet the portion of eche man one after an other, and then worke by the Golden rule, and the Fourthe ſumme will ſhewe you eche mans gaines; as in example.

The parcels of thoſe foure Marchants make in one ſumme 240 lb: ſet that in yͤ firſte place, the gaines in the ſecōd, & the firſt mans portiō of ſtocke in yͤ 3 place, thus.

240 — 3000
30 —

Now multiplie the ſecond by the third, and it will bee 90000, which you ſhall diuide by 240, and there will appeare 375 lb, thus.

240 — 3000
30 — 375

And that is the gaines for the firſt man.

Now for the ſecond man, ſet the 50 lb, that he brought, in the thirde place, and worke as

before

before: and his part will bee 625 li. as this figure sheweth.

240 — 3000
50 — 625

Likewaies for the thirde man set his mony, which was 60 li, and his part of gaines will be 150 li, as here appeareth.

240 — 3000
60 — 750

And so for the fourthe man, if you set his summe which is 1000 li, his gaines will bee 1250 pounde, as the proofe will declare.

24[illegible] — 3000
100 — 1250

Scholer. This I perceyue: but is there anye waye to examine whether I haue well done or no?

Mayster. That muste you doe by one common proofe which serueth to the Golden rule, and all other insuing of the same: and that is this: Chaunge the standings of the numbers, and set the thirde in the firste place, the 4 in the seconde place, and the firste in the thirde place, and then worke by the Golden rule, and if you haue done well, the fourth number nowe will be the same that was the seconde before. As for example, I will take ẏ last worke which was this.

Note this common proofe.

Proofe.

240 — 5000
100 — 1250

100 1250
240

Which to examine, I alter as I saide, thus.

Nowe if I multiplye the second number by the third, and diuide that that amounteth by the first, then will the fourth number bee 3000, which was the seconde before, as you see here:

100 1250
240 3000

which is a token, that I haue well done. But as in a single rule one proofe thus is sufficient, so in a rule where many operations bee, you muste tourne euery of them as I haue done with this one.

Sc. Then for the proofe of the first worke of this rule, I shoulde turne the numbers thus.

30 375
240

And the second, thus.

50 625
240

And for the thirde, thus.

10 750
240

And in eche of them if the working were trewe, the fourth number will be still 3000.

Mayster. Well, now an other example will I put to you, not of gaines, but of losse: for one reason serueth for bothe.

Question. If three Marchantes in one shippe and of one

one fellowship, had bought marchandise, so ẏ the first had layd out 200 li, the seconde 300 li, and the thirde 500 li, and it chaunced by tempest that they did cast ouer boord into the Sea marchandise of the value of 100 pounde, howe muche should eche man beare in this losse? *of losse.*

Scholer. If I shall doe in this as you did in the other question, then muste I ioyne their three portions togither, 200, 300, and 500, which maketh 1000. Then saye I, if 1000 leese 100, then shall 200 leese 20, and 300 shall leese 30, and 500, shall leese 50, as by these three figures it doth appeare plaine.

1000 — 100
200 — 20

1000 — 100
300 — 30

1000 — 100
500 — 50

Mayster. Thus you perceyue the vse of the rule without time. And that you may as well perceiue the same with diuersitie of tyme, I propose this example. *The rule of felovvship vvith time.*

Foure Marchantes made a common stocke, whiche at the yeares ende was encreased to 35145 li. Nowe to knowe what shall be eche mannes portion of gaynes, you muste knowe eche mans stocke and time of continuance. *Question of a banke.*

The first man of these foure layde in 669 li,

whiche hee did take from the stocke agayne, at ende of 10 monethes. The seconde man layde in 810 ℔. for 8 monethes. The thirde layde in 900 ℔. for 7 monethes. And the fourth layde in 1040 ℔. for 12 monethes.

This question shall you examine as you did the other before, sauing that where as in the thirde place of the figure you did set eche mans summe alone, heere you shall set the same being multiplyed by the number of their time: & likewise in the first place of the figure, you shall sette that number whiche amounteth of their whole summes so multiplied by their time, and added into one summe, as thus.

The first mannes summe is 669 ℔, which I multiplie by 10 (that was the number of his time) and it maketh 6690. The second mans summe 810 ℔, multiplied by 8 (whiche was his time) make 6480. The thirde mans summe 900 ℔, multiplied by 7 (for that was his time) yeldeth 6300. The fourth mans summe was 1040 ℔, and his time 12, multiply the one by the other, and it will be 12480.

These foure summes thus multiplied by their time, must be set orderly in the third place of the figure: and in the first place must bee set the whole summe of all foure, whiche is 31950.

and

and the gayne muste bee in the seconde place, whiche is 35145. Now to ende the question, I saye firste: If 31950 did gette 35145, what did 6690 get?

a

Answere, 7359 ℔, as by this figure appeareth.

31950 — 35145
6690 — 7359

Likewise the seconde man had to his part 7128 ℔. the thirde must haue 6930 ℔. And the fourth man shall haue for his part 13728 ℔. as these figures do partly declare.

b

31950 — 35145
6480 — 7128

c

31950 — 35145
6300 — 6930

d

31950 — 35145
12480 — 13728

Scholer. This I like very well: but what proofe is there of this worke?

Mayster. The same that I taught you for the other. Howe be it, there is vsed both for this worke and the other also this maner of proofe, to adde all the portions togither, and if they agree to the whole summe, then seemeth it well done: but this is no sure rule. An other proofe.

Scholer. Yet wil I proue it in this example.

The fowre parcels are these, which if I adde togither, there will amount 35145, and that was the whole summe: so is this rule true heere.	7359 7128 6930 13728 35145

Mayster. And so will it bee still, when the worke is truly done.

Note the imperfection of this kinde of proofe.

But if you liste to see it proued false, take 10000 lb, from the fourth man, and put it to any of the other 3, & then be you sure that you haue not done well, and yet will the prowfe allow it, for the Addition will still be all one.

Scholer. It must needes bee so: but what haue I nowe to learne?

Mayster There are many other excellente partes behinde, of whiche I will not, as nowe, make mention, bicause that without the knowledge of Fractions, they cannot be duly taught, and muche lesse vnderstanded. Therefore will I propose to you two or three questions more, whereby you maye practise the better the feate of the rule of Felowship, and so make an ende for this time.

But this may not bee forgotten, that in all such questions, if the mony be of diuers kindes, you muste by Reduction bring it into one kinde, that is to saye, to the leaste valure that

is

is named in the question. And likewaies shall you doe, if the time be of diuers kindes, as some yeares, some monethes, weekes and daies, you shall make all moneths, weekes or daies, according as the leaste name of tyme in the question is: As for example.

Firste in diuersity of mony. Three companion bought 2000 sheepe, and payde for them 241 li, 13 s, 4 d. of which summe one payde 101 li, 10 s. The seconde payed 82 li. 17 s, 10 d. And the thirde payde 57 li, 5 s, 6 d: Howe manye sheepe muste ech of them haue? Answere: The firste shall haue 840. The seconde 686. And the thirde 474. And that muste you worke thus.

Question of Sheepe.

Solution.

Firste considering that your mony is of diuers denominations, you shall (by Reduction) bring it all into the smallest denomination which is in it, that is to say, pens, and so will the totall summe bee 58000 pens. Now if you turne ech mans mony into pennies also, the firste mannes summe will bee 24360 pens: The seconde mannes summe 19894 d. And the thirde mannes mony will bee 13746 d.

Nowe to knowe howe many sheepe euery man shall haue, set the whole summe of mony

that is 58000 ɗ, in the firste place: and in the seconde place set the number of sheepe, and then orderly in the thirde place set eche mans mony, and then multiplying the third and the seconde summes togither, and diuiding that that amounteth by the firste, there will appeare the number of sheepe that eche man ought to haue: as these three figures we shew.

a		b	
58000	2000	58000	2000
24360	840	19894	686

c	
58000	2000
13746	474

Scholer. Why we you set the monye in the firste place, seeing in the question you saye 2000 sheepe cost 58000 ɗ? and not thus, 58000 ɗ, coste 2000 sheepe.

Mayster. You remember, I taught you at the beginning of this Golden rule, that the firste and third number must bee of one name, and of like things: and euermore the number that the question is asked of, must bee set in the thirde place. Now is the question plainely this: If four men bought 2000 sheepe for 58000 pens, howe many sheepe shall eche man haue?

But

But seing in this question there ought more respect to be had to the summe of mony, than to the summe of the persons, (for in the summes of mony is there proportion toward the sheepe, and not in the number of persons) therefore must we turne the question thus.

If 18000 pence bought 2000 sheepe, howe many did 24360 d. buy? Agayne, howe manie did 19894 d. buye? and howe many bought 13746 pens.

Scholer. I perceyue it reasonable, and so shall I we in all like questions.

Mayster. Euen so. But for easinesse of the worke marke this: When so euer the firste and seconde numbers haue ciphers in their first places, you may bothe in the multiplication and in the diuision leaue out those ciphers, so that you leaue out like manie out of bothe summes, as in this question the firste number 18000 hathe three cyphers, and so hathe the seconde that is 2000: therefore caste awaye their ciphers, and so will the firste number bee 18, and the seconde 2: set them in their places, and worke according to the rule, and you shall perceyue that it will bee all one, sauing that this is the shorter and easier way, as these three figures we shewe. Note.

a

$$58 \quad 2$$
$$24360 \quad 840$$

b

$$58 \quad 2$$
$$19894 \quad 676$$

c

$$58 \quad 2$$
$$13746 \quad 474$$

And this you see is bothe easyer, and also the more certaine way to knowe the answere to this question.

Scholer. Truthe it is as you say: but sir, mee seemeth I might aske a farther question heere, not onely howe many sheepe eche man should haue, but also what euery sheepe cost.

Mayster. That question doeth not onely belong to this rule, but may also bee discussed by Diuision, especially if the questions number bee one onely: as thus. Diuide the totall summe 58000 pens, by 2000 (other 58 by 2, omitting the ciphers) and the quotient will bee 29 pens, that is, 2 ß, 5 d. howe bee it, by this rule you may doe it, and beste when the number of the question doth exceede 1: as if I shoulde aske this question, 2000 sheepe coste 58000 d, howe muche did 20 cost? the shall I set my figure thus.

$$2000 \quad 58000$$
$$20$$

And doing after the rule, there will amounte

580

580 pens : that is 2 ℔, 6 ß, 4 d, the price of one score

But if you will vse that easie way that I did teache you, you may chaunge the first and seconde number, thus.

2 58
10

Yet nowe one question more will I moue (that you may perceiue the vse of al other like) and so make an ende.

There is in a Cathedrall Churche 20 Cannons, and 30 Vicars, those may spende by yeare 2600 ℔, but euery Cannon muste haue to his part 5 times so muche as euery Vicar hathe : howe muche is euerye mannes portion say you? *Question Canons.*

Scholer. I praye you make the answere your selfe, so shall I perceyue best the meanes to answere to such other like.

Mayster. In this question you muste doe as in those that haue diuersitie of time, for here is diuersitie of portions : Therefore shall you multiplye the number of the persons by theyr difference of portion : (as you did in the other by Time.) Then must you multiplie the 20, (whiche is the number of Canons) by 5, (for that is the number of their portion) so will it bee 100 : Then 30, (that is the number of

Vicars)

Vicars) by 1, (that is the number of their por-
tion) and it will be 30: put thoſe two ſummes togither, and they make 130: then ſay thus: If 130 ſpende 2600 ℔, what may 100 ſpende? The rule ſheweth 2000 ℔.

Vicars. Againe for Vicars: If 130 ſpende 2600 pounde, what may 30 ſpende? Anſwer 600 ℔, as theſe figures ſhewe.

130	2600	ǁ	130	2600
100	2000	ǁ	30	600

But if euery Canon ſhoulde haue ſo often times 4 ℔, as the Vicar ſhould haue 3 ℔, then ſhould I multiply 20 by 4, (that were 80) and 30 by 3 (that were 90) and then bothe were 170. Then ſhould the figures be ſet thus.

	℔. ß. d.			℔. ß. d.
170	2600	ǁ	170	2600
80	1223,10,7	ǁ	90	1376,9,5,

But this ſort is to harde for you, by reaſon of the fractions, therefore I will let it reſte to that place. And by this rule you ſee what the 20 Cannons maye ſpende, whiche ſumme if you diuide by 20, you ſhall ſee eche Cannons portion: and ſo of ẏ Vicars, if you diuide their ſumme by 30, the quotient will declare euery Vicars portion.

The

The ſeconde Dialogue.

The accompting by Counters.

Mayſter.

NOwe that you haue learned the common kindes of Arithmetike with the pen, you ſhall ſee the ſame arte in Counters: which feate dothe not onely ſerue for them that cannot write and reade, but alſo for them that can doe both, but haue not at ſome times their penne or tables readie with them.

This ſorte is in two formes commonlye: The one by lines, and the other without lines. In that that hath lines, the lynes do ſtande for the order of places: and in that that hathe no lines, there muſt bee ſet in their ſteade ſo many counters as ſhall neede, for eche line one, and they ſhall ſupply the ſteade of the lines.

Scholer. By examples I ſhoulde better perceyue your meaning.

Mayſter.

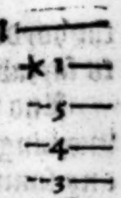
M. For example of — 100000
the Lynes, ſo heere — 10000
you ſee vj. lines, whi- ✱ 1000
che ſtande for vj. pla- — 100
ces, ſo that the nether- — 10
moſte ſtandeth for the — 1
firſt place, and the next about it for the ſecond, and ſo vpwarde, till you come to the higheſt, whiche is the ſixt line, and ſtandeth for the ſixt place.

Nowe what is the value of euery place or line you maye perceyue by the figures whiche I haue ſet on them, whiche is according as you learned before that in Numeration of figures by the pen: for the firſt place is the place of vnits or ones, and euery counter ſet in that line betokeneth but one: and the ſeconde line is the place of 10, for euerye counter there ſtandeth for 10. The thirde line the place of hundreds, the fourth of thouſands: and ſo forth.

Scholer. Sir, I doe perceyue that the ſame order is heere of lines, as was in the other figures by places, ſo that you ſhall not neede longer to ſtande about Numeration, excepte there be any other difference.

Mayſter.

M. If you doe vnderstande it, then how will you set 1543?



Scholer. Thus as I suppose.

Ma. You haue set the places truly, but your figures bee not meete for this vse: for the meetest figure in this behalfe is the figure of a counter, round, as you see here, where I haue expressed that same summe.

Scholer. So that you haue not one figure for 2, nor 3, nor 4, and so forthe, but as manye digits as you haue, so many counters you set in the lowest line: and for euery 10 you set one in the seconde line: and so of other. But I knowe not by what reason you set that one counter for 500 betweene two lines.

Mayster. You shall remember this, that whensoeuer you neede to set downe 5, 50, or 500, or 5000, or so forth any number whose numerator is 5, you shall set one counter for it in the next space aboue the line that it hathe his denomination of: as in this example of that 500, bicause the numerator is 5, it muste bee set in a voyde space: and bicause the denominator is hundred, I knowe that his place is the

the voyde space next aboue hundredes, that is to say, aboue the thirde line.

And farther you shall marke, that in all working by this sorte, if you shall set downe any summe betweene 4 and 10, for the firste part of that number, you shall set downe 5, and then so many counters more, as there rest nūbers aboue 5. And this is true bothe of digittes and articles. And for example I will set downe this sūme 287965, which summe if you marke well, you neede none other examples for to learne the Numeration of this forme.

But this shall you marke, that as you did in the other kindes of Arithmetike, set a pricke in the places of thousandes, in this worke you shall set a Starre, as you see before.

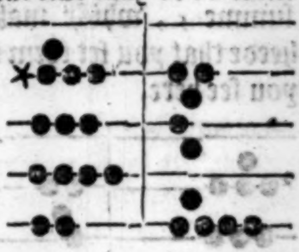
Scholer. Then I perceyue Numeration: but I pray you, how shall I we in this arte to adde two summes or more togither?

Addition

ADDITION.

Mayſter.

The eaſieſt way in this arte, is to adde but two ſummes at ones togither: howbeit, you may adde more, as I will tell you anone. Therefore when you will adde two ſummes, you ſhall firſte ſet downe one of them, it forceth not whiche, and then by it drawe a line croſſe the other lines. And afterwarde ſet downe the other ſumme, ſo that that line may be betwene them: as if you woulde adde 2659 to 8342, you muſte ſette youre ſumes as you ſee here.



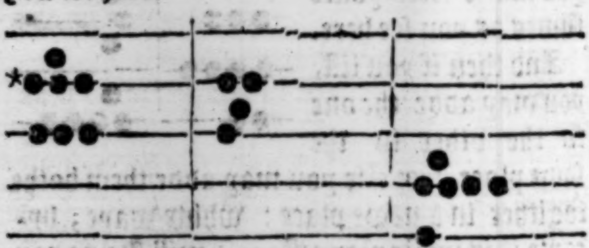
Addition of two ſummes.

And then if you liſt, you may adde the one to the other in the ſame place: or elſe you may adde them bothe togither in a newe place: which waye, bycauſe it is moſte playneſſe, I will ſhewe you firſte.

Therfore will I beginne at the vnits whiche in the firſte ſumme is but 2, and in the ſeconde ſumme 9, that maketh 11. Thoſe doe I take

vp, and for them I ſet 11 in the newe roome, thus.

Then doe I take vp all the articles vnder a hundred, which in the firſte ſumme are 40, and in the ſeconde ſumme 50, that maketh 90: or you may ſay better, that in the firſt ſumme there are 4 articles of 10, and in the ſeconde ſumme 5, which maketh 9, but then take heede that you ſet them in their right lines, as you ſee here.



Where I haue taken away 40 from the firſte ſumme, and 50 from the ſeconde, and in their ſteede I haue ſet 90 in the thirde roome, which I haue ſet plainely, that you myght

well

well perceyue it: howe be it, seeing that 9. with the 10 that was in the thirde roome alrea-dy, doth make 100, I might better for those 6 Counters set 1 in the third line, thus.

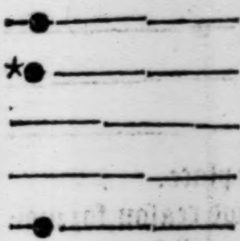
For it is al in one sum as you may see, but it is best neuer to set 5 counters in any line, for yͭ may be done with one counter in a higher place.

Scholer. I iudge that good reason, for many are vnneedfull where one will serue.

Mayster. Well, then will I adde forth of hundreds: I finde 3 in the firste summe, and 6 in the seconde, whiche maketh 900, them doe I take vp, and set in the thirde roome, where is one hundred alreadie, to which I put 900, and it will be 1000, therefore I set one counter in the fourth line for them all, as you see heere.

Then adde I the thousands together, which

in the first summe are 8000, and in the seconde 2000, that maketh 10000: then doe I take vp from those two places, and for them I set one counter in the fift line, and then appeareth



as you see to bee 11001, for so manye dothe amounte of the Addition of 8342 to 2659.

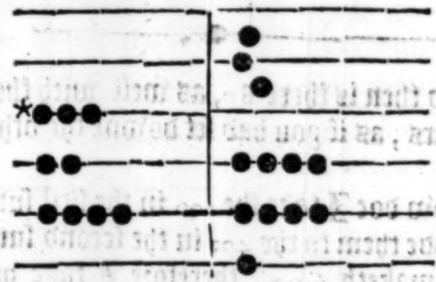
To adde 8 ummes to- ither.

Scholer. Syr, this I doe perceyue: but howe shall I sette one summe to an other, not chaunging them to a thirde place.

Mayster. Marke well how I do it: I will adde togither 65436 and 3245, which first I set downe thus.

Then doe I begin with the smallest, which in the first summe is 5, that doe I take vp, and woulde

woulde put to the other 5 in the second summe, sauing that two counters can not bee set in a voyde place of 5 but for them bothe I must set 1 in the seconde line, whiche is the place of 10: therefore I take vp the 5 of the first summe, and the 5 of the seconde, and for them I sette 1 in the seconde line, as you see heere.



Then doe I likewayes take vp the 4 counters of the first summe and seconde line (which make 40) and adde them to the 4 counters of the same line, in the seconde summe, and it maketh 80. But (as I sayde) I maye not conueniently set aboue 4 Counters in one line, therefore to those 4 that I tooke vp in the firste summe, I take one also of the seconde summe, and then haue I taken vp 50, for whiche 5 Counters I set downe one in the space ouer the seconde line, as heere dothe

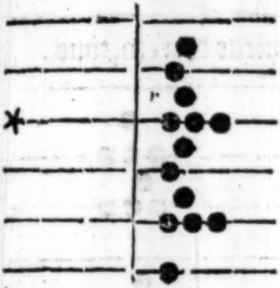
appeare.

And then is there 80, as well with those 4 counters, as if you had set downe the other 4 also.

Now doe I take the 200 in the first summe, and adde them to the 400 in the second summe, and it maketh 600, therefore I take vp the 2 counters in the first summe, and 3 of them in the seconde summe, and for them 5, I set 1 in the space aboue, thus.

Then

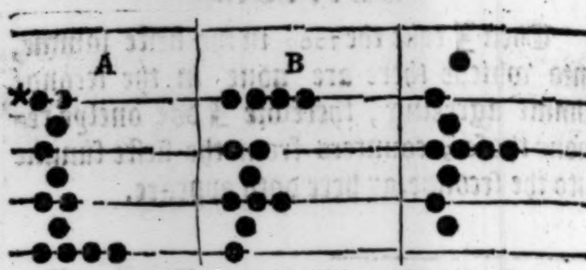
Then I take the 3000 in the firste summe, unto whiche there are none in the seconde summe agreeing, therefore I doe onelye remoue those 3 counters from the firste summe into the seconde, as here doth appeare.



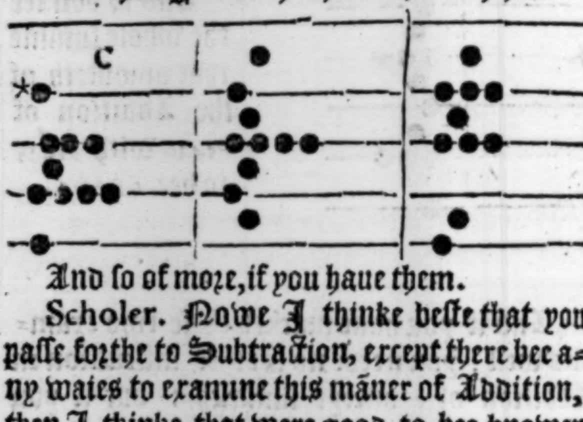
And so you see the whole summe that amounteth of the Addition of 65436 with 3245, to bee 68681.

And if you haue marked these two examples well, you neede no farther instruction in Addition of 2 onelye summes: but if you haue more than two summes to adde, you may adde them thus.

Firste adde two of them, and then adde the thirde and the fourthe, or more if there bee so many: as if I woulde adde 2679 with 4286 and 1391. Firste I adde the two firste summes, thus.



And then I adde the thirde thereto, thus.



And so of more, if you haue them.

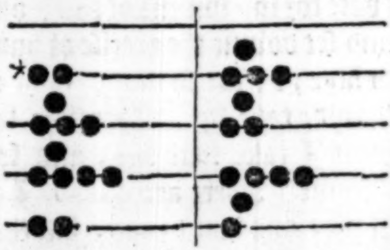
Scholer. Nowe I thinke beste that you passe forthe to Subtraction, except there bee any waies to examine this mãner of Addition, then I thinke that were good to bee knowen nexte.

Maister. There is the same proofe here that is in the other Addition by the penne, I meane Subtraction, for that onely is a sure way: but considering that Subtraction muste be firste knowen, I will first teache you the arte of Subtraction, and that by this example.

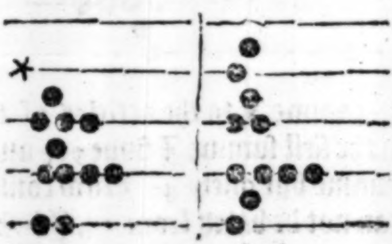
Subtraction

SVBTRACTION.

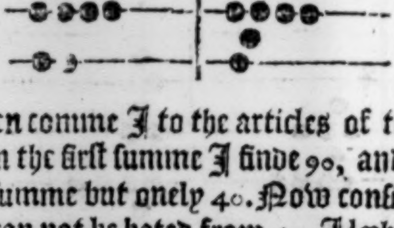
I Woulde subtract 2892 out of 8746. These summes muste I set downe as I did in Addition: but here it is beste to sette the lesser number firste, thus.



Then shall I beginne to subtract the greatest numbers first (contrary to the use of the penne) that is the thousandes in this example: therefore I finde amongest the thousandes 2, for which I withdrawe so many from the seconde summe (where are 8) and so remaineth there 9, as this example sheweth.



Then doe I likewaies with the hundredes, of which in the firste summe I finde 8, and in the seconde summe but 7, out of which I can not take 8, therefore this muste I doe: I muste looke howe muche my summe differeth from 10, which I finde heere to bee 2, then muste I bate for my summe of 800, one thousande, and set downe the excesse of hundredes, that is to saye, 2, for so much 1000 is more than I shoulde take vp. Therefore from the firste summe I take that 800, and from the seconde summe (where are 6000) I take vp one thousande, and leaue 5000, but then set I downe the 200, vnto the 700 that are there already, and make them 900, thus.

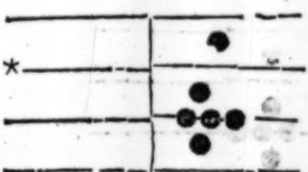


Then comme I to the articles of tennes, where in the first summe I finde 90, and in the second summe but onely 40. Now considering that 90 can not be bated from 40, I looke howe muche

muche that 90 doth differ from the next summe aboue it, that is 100, or else (which is all to one effecte) I looke howe muche 9 doth differ from 10, and I finde it to bee 1, then in the steade of that 90, I doe take from the seconde summe 100: but considering that it is 10 to much, I set downe 1 in the nexte line beneath for it, as you see heere.

Sauing that heere I haue set one counter in the space, in steade of 5, in the next line.

And thus haue I subtracted all saue two, whiche I muste bate from the 6 in the seconde sūme, and there will remaine 4, thus.

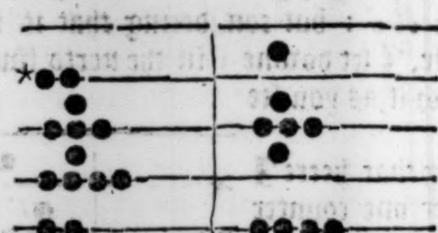


So ẏ if I subtract 2892 from 8746, the remayner will bee 5854.

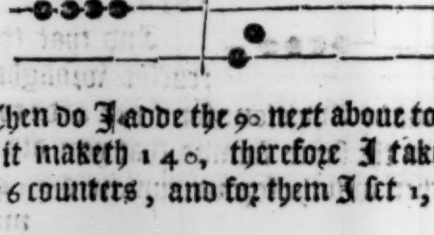
And that this is truelye wrought, you may proue by Addition: for if you adde to this remayner the same summe that you did subtracte, then will the former summe 8746, amount

amount againe.

A proofe. Scholer. That will I proue: and first I set the summe that was subtracted, which was 2892, and then the remayner 5854, thus.

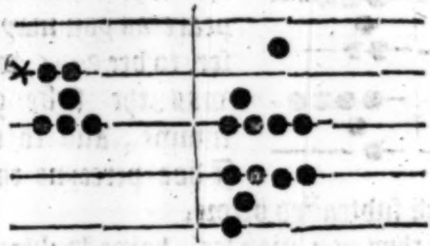


Then doe I adde first the 2 to 4, which maketh 6: so take I vp 5 of those counters, and in their steade I set 1 in the space, and 1 in the lowest line, as here appeareth.

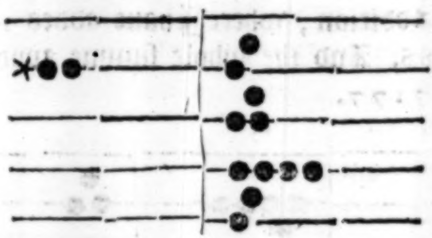


Then do I adde the 90 next aboue to the 50, and it maketh 140, therefore I take vppe those 6 counters, and for them I set 1, to the hun-

hundꝛeds in the thirde line, and foure in the seconde line, thus.

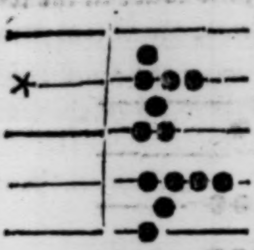


Then do I come to the hundꝛeds, of whiche I finde 8 in the firſt ſumme, and 9 in the ſeconde, that maketh 1700: therefoꝛe I take vp thoſe 9 counters, and in their ſteade I ſet 1 in the fourth line, and 1 in the ſpace nexte beneath, and 2 in the thirde line as you ſee here.



Then is there left in the firſte ſumme but only 2000, which I ſhall take vp from thence, and ſet in the ſame line in the ſeconde ſumme,

to



to the one that is there alreadie: and then will the whole ſumme ap‑peare as you may well ſee, to bee 8746, whiche was the firſte groſſe ſumme, and therefore I doe perceyue that I had well ſubtracted before.

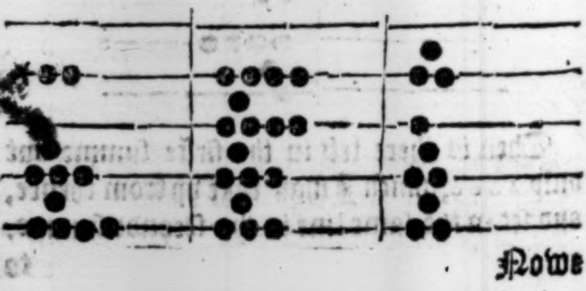
And thus you may ſee, howe Subtraction may be tryed by Addition.

Scholer. I perceyue the ſame order heere with Counters, that I learned before in fygures.

Mayſter. Then let mee ſee howe can you trie Addition by Subtraction.

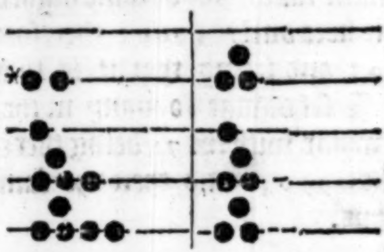
Scholer. Firſt I will ſet forth this example of Addition, where I haue added 2189, to 4988. And the whole ſumme appeareth to bee 7177.

A proofe of Addition, by Subtraction.



Nowe

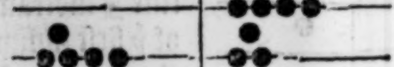
Now to trie whether that summe bee well added or no, I will subtracte one of the firste two summes from the thirde, and if I haue well done, the Remainer will be like that other summe, as for example. I will subtracte the firste summe from the third, which I set thus, in their order.



Then doe I subtract 2000 of the first summe from the seconde summe, and then remayneth there 5000, thus.

Then in the thirde line I subtracte yͤ 100 of yͤ first summe from the seconde summe, where is onelye 100 also: and then in the thirde line resteth nothing, as you may see in the page following.

Then in the second line with his space ouer him, I finde 80, which I shoulde subtracte from the other summe, then seeing there are but onelye 70, I muste take it out of some higher summe, which is here onely 5000: therefore I take vp 5000: and seeing that it is to much by 4920, I set downe so many in the seconde roume, which with the 70 being there already doe make 4990, and then the summes doe stande thus.



Yet remaineth there in the firste summe 9 to be bated from the seconde summe, wherein that place of vnits doth appeare onely 7: then must I bate a higher summe, that is to say 10, but seeing that 10 is more than 9 (which I shoulde abate

abate) by 1, therfore shall I take vp one counter from the seconde line, and set downe ỹ same in the firste or lowest line, as you see here.

And so haue I ended this worke, and the summe appeareth to be the same which was the seconde summe of myne Addition, and therefore I perceyue I haue well done.

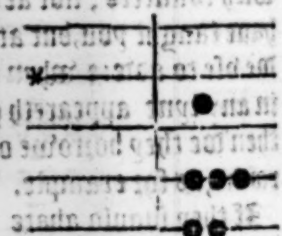
Mayster. To stande longer about this, it is but folly, except that this you maye also vnderstande, that many do beginne to subtract with counters, not at the highest summe as I haue taught you, but at the nethermost, as they do vse to adde: & when the summe to be abated in any lyne appeareth greater than the other, then do they borrowe one of the nexte higher rome, as for example. An other way of Addition.

If they should abate 1846 from 2378, they sette the summes thus.

Firste they take 6, which is in the lower lyne, and his space, from 8 in the same romes

in the seconde summe, and yet there remaineth 2 counters in the lowest lyne. Then in the seconde lyne muste 4 bee subtracted from 7, and so remayneth there 3. Then 800 in the thirde lyne, and his space, from 300 of the seconde summe, can not bee, therefore we they bate it from a higher rowme, that is from 1000: and bycause that 1000. is to much by 200, therefore muste I set downe 200 in the thirde lyne, after I haue taken vp 1000 from the fourthe lyne. Then is there yet 1000 in the fourthe lyne of the firste summe, which if I withdrawe from the seconde summe, then dothe all the figures stande in order, thus.

So that (as you see) it differeth not greately whether you beginne Subtraction at the higher lynes, or at the lower.



Howe be it, as some men like the one way beste, so some lyke the other: therefore you nowe knowing bothe, may vse which you liste.

BUt nowe touching Multiplication : you shall set youre numbers in two roomes (as you did in those two other kindes) but so that the multiplyer be set in the firste roume, then shall you begin with the highest numbers of the seconde roome, and multiplie them first, after this sorte. Take that ouermoste line in youre first working, as if it were the lowest line, setting on it some moueable marke (as you lyste) and looke howe many counters bee in him, take them vp, and for them sette downe the whole multiplyer so many tymes as you tooke vppe counters : reckening (I saye) that line for the Units. And when you haue so done with the highest number, then come to the next line beneath, and doe euen so with it, and so with the nexte, tyll you haue done all. And if there bee anye number in a space, then for it shall you take the multiplyer 5 tymes : and then muste you recken that line for the Units, which is nexte beneath that space. Or else after a shorter waye, you shall take onely halfe the multiplyee, but then shall you take the

lyne next aboue that space for the line of vnits. But in suche workinge, if by chaunce your multiplyer bee an odde number, so that you can not take the halfe of it iustlye, then must you take the greater halfe, and set downe that, as if that it were the iust halfe: and farther you shall set one Counter in the space beneath that line, whiche you recken for the line of vnits, or else onelye remoue forwarde the same that is to be multiplyed.

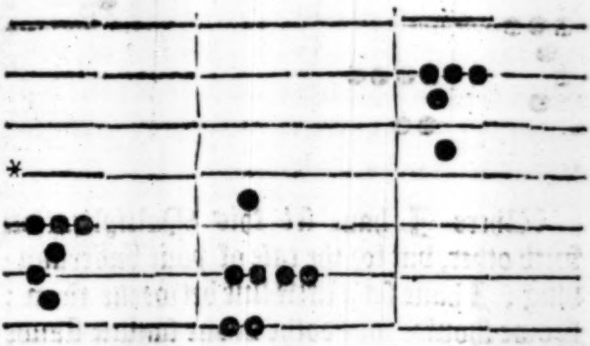
Scholer. If you set forth an example hereto, I thinke I shall perceyue you.

Mayster. Take this example: I woulde multiplie 1542 by 365, therfore I set the numbers thus.

Then firste I beginne at the 1000 in the highest roume, as if it were the firste place, and I take it vp, setting downe for it so often (that is once) the multiplyer, whiche is 365, thus as you see heere: where, for the one counter taken vp from the fourthe line, I haue sette downe other 6, whiche make the

summe

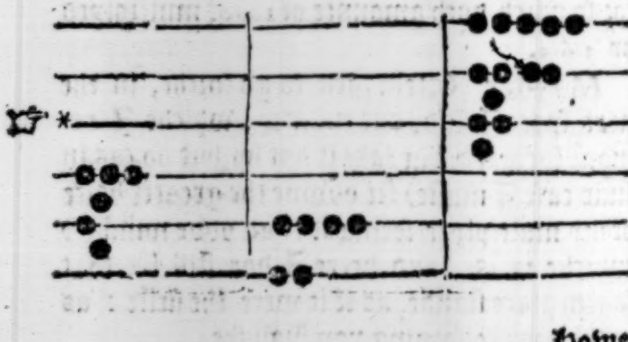
ſumme of the multiplier, reckening that fourth line as if it were the firſte, whiche thing I haue marked by the hande ſet at the beginning of the ſame.



Scholer. I perceyue this well, for in deede this ſumme that you haue ſet down is 365000: for ſo much doth amounte of 1000, multiplyed by 365.

Mayſter Well, then to go forthe, in the next ſpace I finde one counter, whiche I remoue forward, but take it not vp, but do (as in ſuche caſe I muſte) ſet downe the greater halfe of my multiplyer (ſeeing it is an odde number) whiche is 182, and heere I doe ſtill let that fourth place ſtande, as if it were the firſte: as in the page following you ſhall ſee.

Where I haue set this Multiplication with other, but for the ease of your vnderstanding, I haue set a little line betweene them: Nowe shoulde they bothe in one summe stande thus.



Nowe

How be it, an other forme to multiplye suche coūters in space, is this: Firste to remoue the finger to the lyne nexte beneth that space, and then to take vp that Counter, and to set downe the multiplyer fiue tymes: as here you see.

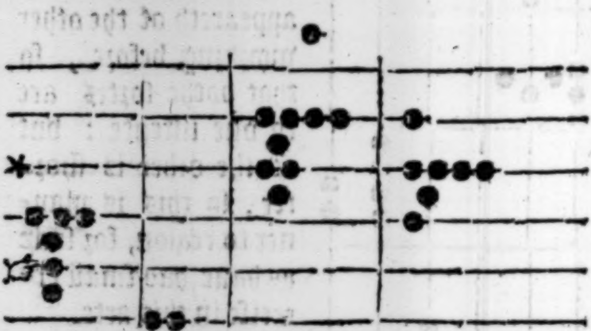
An other forme of Multiplication.

Whiche summes if you doe adde togither into one summe, you shall perceyue that it wyll bee the same that appeareth of the other working before, so that bothe sortes are to one intente: but as the other is shorter, so this is playner to reason, for suche as haue had small exercise in this arte.

Notwithstādīg you

may adde them in your minde before you sette them downe: as in this example you myght haue sayde, 5 tymes 300 is 1500, and 5 times 60 is 300: Also 5 times 5 is 25, which all put togither, doe make 1825, which you may at one tyme set downe if you liste.

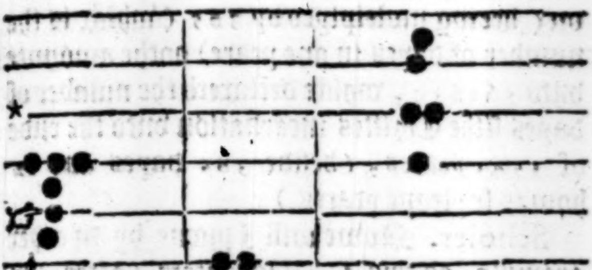
But now to go forth, I muste remoue the hande to the nexte counters which are in the seconde lyne, and there muste I take vp those 4 counters, setting downe for them my multiplyer 4 tymes seuerally, or else I may gather that whole summe in my mynde first, and then set it downe: as to say, 4 times 300 is 1200: 4 tymes 60 are 240: and 4 times 5 make 20, that is in all 1460, that shall I set downe also, as here you see.



Which if I ioyne in one summe wt the for-

mer

mer numbers, it will appeare thus.



Then to ende this Multiplication, I remoue the finger to the lowest lyne, where are onelye 2, them doe I take vppe, and in their steede doe I set downe twice 365, that is 730, for which I sette one in the space aboue the thirde line for 500, and 2 more in the thirde lyne with that one that is there all ready, and the rest in their order, and so haue I ended the whole summe, thus.

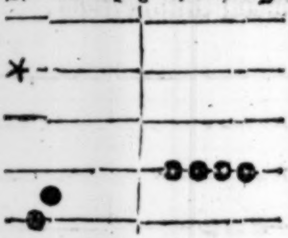
Whereby you see, that 1542 (which is the number of yeares sithe Christes incarnation (beeing multiplyed by 365 (which is the number of dayes in one yeare) dothe amounte vnto 562830, which declareth the number of dayes sithe Christes incarnation vnto the ende of 1542 yeares, (beside 385 dayes and 12 houres for leape yeares.)

The summe of the daies sith Christs incarnation

Scholer. Nowe will I proue by an other example, as this : 40 labourers (after 6 d. the day for ech man) haue wrought 28 dayes: I woulde knowe what their wages dothe amounte vnto.

A question of wages.

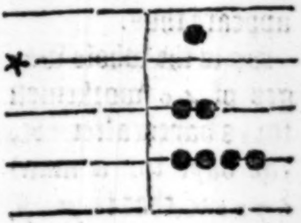
In this case muste I worke doubelye: firste I muste multiplye the number of the labourers by the wages of a manne for one day, so will the charge of one daye amounte. Then secondarily shall I multiply the charge of one daye by the whole number of dayes, and so will ỹ whole summe appeare: first therefore I shall set the summes thus.



Where in the firste place is the multiplyer (that is one dayes wages for one man) and in the second space is set

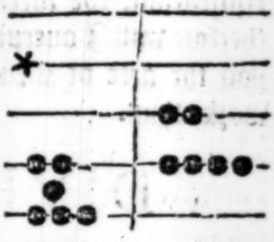
set the number of ẏ workmen to be multiplied.

Then saye I: 6 times 4 (reckening that seconde line of the line of Units) maketh 24, for whiche summe I shoulde set 2 counters in the thirde line, and 4 in the seconde, therefore doe I set 2 in the thirde line, and let the 4 stande still in the seconde line thus.

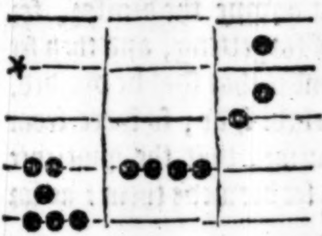


So appeareth the whole dayes wages to be 240 d, that is 20 ß.

Then doe I multiplye agayne the same summe by the number of dayes, and firste I set the numbers thus.



Then bicause there are counters in diuerse lines, I shall begin with the highest, and take them vp, setting for them the multiplyer so many times as I toke vp counters, that is twise, then will the sum stand thus.



Then

Then come I to the seconde line, and take vp those 4 counters, setting for them the multiplyer foure tymes, so will the whole summe appeare thus.

So is the whole wages of 40 workemen for 28 dayes (after 6 d. eche daye for a man) 6720 d. that is 560 ß, or 28 pounde.

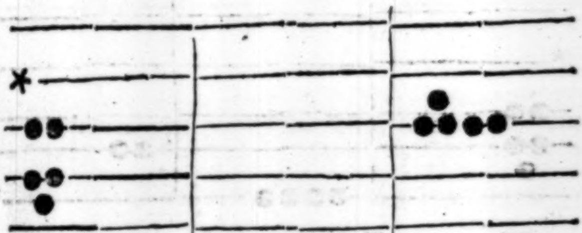
Mayster. Now if you woulde proue multiplication, the surest waye is by Diuision: therfore will I ouerpasse it, till I haue taught you the arte of Diuision, which, you shall worke thus.

DIVISION.

First set downe the diuisor, for feare of forgetting, and then set the number that shal be diuided, at the right syde, so farre from the Diuisor, that the quotiente may be set betwene them: as for example.

If 225 sheepe cost 45 li, what did euerye

sheepe coste? To knowe this, I shoulde diuide the whole summe, that is 45 ℔, by 225, but that cannot bee, therefore must I firste reduce that 45 ℔, into a lesser denomination, as into shillings: then I multiply 45 by 20, and it is 900: that summe shall I diuide by the number of sheepe, which is 225, these two numbers therefore I set thus.



Then beginne I at the highest lyne of the diuidente, and seeke howe often I maye haue the diuisour therein, and that may I doe foure times: then say I, foure times 2 are 8, which if I take from 9, there resteth but 1, thus.

And bycause I founde the diuisour 4 times in the diuidente, I haue sette as you see, 4 in the middle rowme, whiche is the place of the quotient: but nowe must I take the rest of the diuisour as often out of the remayner, therefore come I to the seconde line of the diuisour, saying: 2 foure tymes make 8, take 8 from 10, and there resteth 2, thus.

Then come I to the lowest number, which is 5, and multiplie it 4 times, so is it 20, that take I from 20, & there remayneth nothing, so that I see my quotient to bee 4, which are in value shillings, for so was the diuident: and thereby I knowe that if 225 sheepe did cost 45 ℔, euery sheepe cost 4 ß.

Example of vvages.

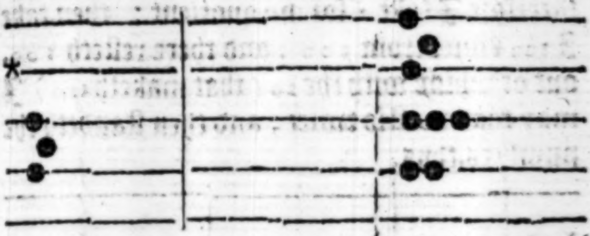
Scholer. This can I doe, as you shall perceiue by this example. If 160 souldiers do spēd euery moneth 68 ℔, what spendeth eche man?

First bycause I can not diuide the 68, by 160, therfore I will turne the pounds into pen-

nies

nies by multiplication, so shall ther be 16320 d.

Nowe muste I diuide this summe by the number of souldiors, therefore I sette them in order, thus.



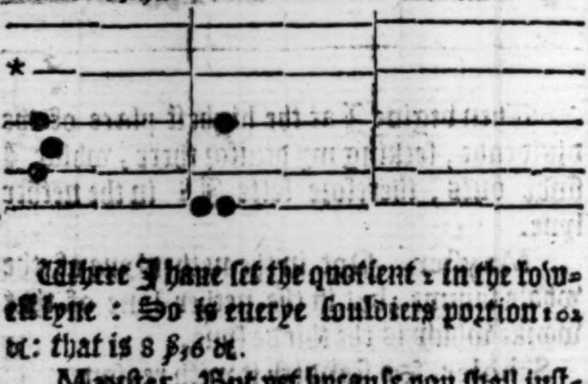
Then begine I at the highest place of the diuidende, seeking my diuisor there, which I finde ones, therefore sette I 1 in the nether lyne.

Mayster. Not in the nether lyne of the whole summe, but in the nether lyne of that worke which is the thirde lyne.

Scholer. So standeth it with reason.

Maister. Then thus do they stande.

Then seeke I againe the reste, howe often I maye finde my diuisour: and I see that in the 300 I might finde 100 three tymes, but then the 60 will not bee so often founde in 20, therefore I take 2 for my quotient: then take I 100 twice from 300, and there resteth 100, out of which with the 20 (that maketh 120) I may take 60 also twice, and then standeth the numbers, thus.



Where I haue set the quotient 2 in the lowest lyne: So is euerye souldiers portion 102 d: that is 8 ß, 6 d.

Mayster. But yet bycause you shall iustlye perceyue the reason of Diuision, it shall bee good that you doe sette your diuisor still against those numbers from which you doe take it, as by this example I will declare.

Example of purchase

If the purchase of 200 acres of grounde did coste 290 pounde, what did one acre cost?

Firste will I tourne the poundes into pen-

nyes,

nies, so will there bee 69600 pens. Then in setting downe these numbers, I shall doe thus. First set the diuidend on the right hande as it oughte, and then the diuisor on the lefte hande agaynste those numbers from which I intende to take him firste, as heere you see, where I haue set the diuisor two lines higher thā is his owne place.

Scholer. This is like the order of Diuision by the penne.

Mayster. Truthe you say: and nowe muste I set the quotient of this worke in the thirde lyne, for that is the lyne of Units in respect to the diuisor in this worke.

Then I seeke howe often the diuisor maye bee founde in the diuidente, and that I finde 3 tymes, then sette I 3 in the thirde lyne for the quotient, and take awaye that 60000 from the diuidend, and farther I doe sette the diuisor

one line lower, as you see here.

And then seeke I how often the diuisor will bee taken from the number agaynste it, which will bee 4 times, and 1 remaining.

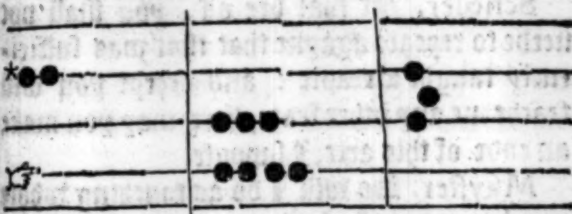
Scholer. But what if it chaunce that when the diuisor is so remoued, it can not bee ones taken out of the diuident agaynste it?

Mayster. Then muste the diuisour be sette in an other lyne lower.

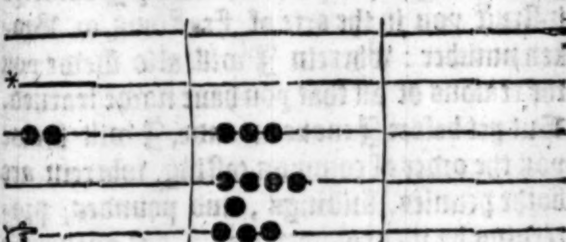
Scholer. So was it in Diuision by yͤ penne, and therefore was there a cipher set in the quotient: but how shall that bee noted here?

Mayst. Here needeth no token, for the lines we represent the places: onely looke that you set your quotient in that place, which standeth for vnits in respect of the diuisour. But nowe to returne to the example. I finde the diuisor 4 tymes in the diuident, and 1 remayning, for 4 tymes 2 make 8, which I take from 9, & there

resteth 1, as this figure followyng sheweth: and in the middle space for the quotient I set 4 in the second line, whiche is in this worke the place of vnits.



Then remoue I the diuisour to the nexte lower line, and seeke how oftē I may haue it in the diuident, whiche I may doe heere 8 times iust, and nothing remayne, as in this fourme,



Where you maye see, that the whole quotient is 348 d, that is 29 ſ, whereby I knowe that so muche cost the purchase of one acre.

Scholer. Nowe resteth the proues of Multiplication, and also of Diuision,

Mayster. Their best proues are eche one by the other: for multiplication is proued by Diuision, and Diuision by Multiplication, as in the worke by the pen you learned.

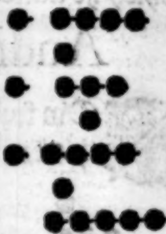
Scholer. If that bee all, you shall not neede to repeate agayne that that was sufficiently taught alreadie: and except you will teache me any other feate, here may you make an ende of this arte, I suppose.

Mayster. So will I do as touching whole number, and as for broken number, I will not trouble your witte with it, till you haue practised this so well, that you bee full perfect, so that you neede not to doubt in any poynt that I haue taught you, and then may I boldelye instruct you in the arte of Fractions or Broken number: wherein I will also shewe you the reasons of all that you haue nowe learned. But yet before I make an ende, I will shewe you the order of common casting, wherein are bothe pennies, shillings, and poundes, proceeding by no grounded reason, but onely by a receyued forme, and that diuerslye of diuerse men: for the Marchantes vse one fourme, and Auditours an other.

The reasons of all the formier rules.

Mar-

But firſte for Marchauntes fourme, marke this example here, in whiche I haue expreſſed this ſumme 198 ℔, 19 ß 11 d, So that you may ſee, that the loweſt line ſerueth for pennies, the nexte aboue for ſhillings, the thirde for poundes, and the fourthe for ſcores of poundes.



And further you may ſee, that the ſpace betwene pens and ſhillings maye receyue but one counter (as all other ſpaces likewayes do) and that one ſtandeth in that place for 6 d.

Likewayes betweene the ſhillings and the poundes, one counter ſtandeth for 10 ß.

And betweene the poundes and 20 ℔. one counter ſtandeth for 10 ℔.

But beſide thoſe you maye ſee at the lefte ſide of ſhillings, that one counter ſtandeth alone, and betokeneth 5 ß.

So agaynſt the poundes, that one counter ſtandeth for 5 ℔. And agaynſt the 20 poundes, the one counter ſtandeth for 5 ſcore poundes, that is 100 pounde, ſo that euery ſide counter is

is 5 times so muche as one of them agaynst which he standeth.

Auditours accompt.

Auditours accompt.

NOw for the accompt of Auditours, take this example.

Where I haue expressed the same summe 198 ℔ 19 ß 11 d.

But here you see the pennies stand towards the right hande, and the other increasing orderly towarde the left hande.

Againe you maye see that Auditours will make 2 lines (yea and more) for pennies, shillings, and all other values, if their summes extende thereto. Also you see that they set one counter at the right ende of eche rowe, which so set there, standeth for 5 of that roume: and on the left corner of the rowe it standeth for 10 of the same rowe.

But now if you would adde other subtract after anye of bothe those sortes, if you marke the order of the other feate whiche I taught you, you may easily doe the same here without

muche

much teaching : for in Addition you muste firste sette downe one summe, and to the same sette the other orderly, and like manner if you haue many : but in Subtraction you muste set downe firste the greatest summe, and from it muste you abate the other, euery denomination from his due place.

Scholer. I doe not doubt but with a little practise I shall attayne these bothe : but howe shall I multiply and diuide after these formes?

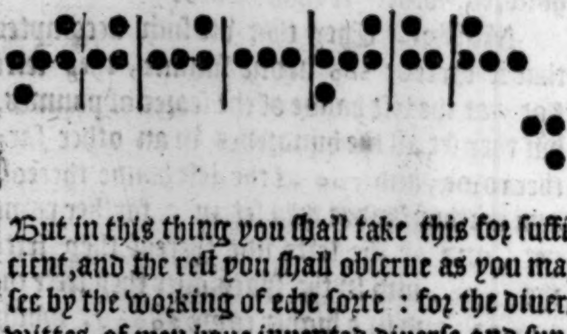
Mayster. You can not duelye doe none of bothe by these sortes, therefore in such case you muste resort to your other artes.

Scholer. Sir, yet I see not by these sortes how to expresse hundredes, if they exceede one hundred, nother yet thousandes.

Maister. They that vse such accomptes that it exceede 200 in one summe, they sette not 5 at the left hande of the scores of poundes, but they set all the hundredes in an other farther rowe, and 500 at the left hande thereof, and the thousandes they set in a farther rowe yet, and at the lefte side thereof they sette the 5000, and in the space ouer they sette the 10000, and in a higher rowe 20000, which all I haue expressed in this example, which is 97869 li, 12 s, 9 d, ob, q. Ninety seuen

thousande, eight hundred, three score and nyne pounde, twelue shillings and nine pens halfe-peny farthing, for I had not tolde you before, where, nother how you shoulde set dawne farthings, which (as you see here) ☞ must be set in a voyde space sideling benethe the pennyes: for q̄ one counter, for ob, 2 counters: for ob, q̄, 3 counters: and more there can not be: for 4 farthings we make 1 d. which must bee in his dewe place.

And if you desire the same summe after Auditours manner: Lo here it is.



But in this thing you shall take this for sufficient, and the rest you shall obserue as you may see by the working of eche sorte: for the diuers wittes of men haue inuented diuerse and sundry wayes, almoste vnnumerable.

The

THE ARTE OF NVMBRING ON THE HANDE.

But one feate I shall teache you, which not onely for the strangenesse & secretnesse is much pleasaunt, but also for the good commoditie of it, right worthy to bee well marked.

This feate hath bin vsed aboue 2000 yeares at the leaste, and yet was it neuer commonlye knowen, especially in Englishe it was neuer taught yet. This is the arte of numbring on the hande, with diuers gestures of the fingers, expressing any summe conceiued in the minde. And firste to beginne.

If you will expresse any summe vnder 100, you shall expresse it with your lefte hande: and from 100 vnto 1000, you shall expresse it with your right hande, as here orderly by this Table following you may perceyue.

(∴)

Here followeth the Table of the arte of the hande.


1
2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
2000
3000
4000
5000
6000
7000
8000
9000

1 In whiche (as you may ſee) 1 is expreſſed 1
by the little finger of the left hand, cloſely and
harde crooked.

2 Is declared by like bowing of the wed- 2
ding finger (which is the next to the little fin-
ger) togither with the little finger.

3 Is ſignified by the middle finger, bowed 3
in like maner with thoſe two.

4 Is declared by the bowing of the middle 4
finger, and the ring finger or wedding finger,
with the other all ſtretched forth.

5 Is repreſented by the middle finger one- 5
ly bowed.

And 6 by the wedding finger onely croo- 6
ked: and this you maye marke in theſe a cer-
tayne order. But nowe 7, 8, and 9, are ex-
preſſed with the bowing of the ſame fingers,
as are 1, 2 and 3, but after another fourme,

For 7 is declared by the bowing of the little 7
finger as is 1, ſaue that for 1 the finger is
claſped in, harde and rounde, but for to expreſſe
7, you ſhall bow the middle ioynt of the little
finger onelye, and holde the other ioyntes
ſtraight.

Scholer. If you will giue mee leaue to
expreſſe it after my rude manner, thus I vn-
derſtande your meaning: that one is expreſſed

by

by crooking in the little fynger, like the head of a bishops bagle: and 7 is declared by the same finger bowed like a gibbet.

Mayster. So I perceyue you vnderstand it.

8 Then to expresse 8, you shall bowe after the same maner bothe the little finger, and the ring finger.

9 And if you bowe likewise with them the middle finger, then doth it betoken 9.

10 Nowe to expresse 10, you shall bowe youre forefinger rounde, and set the ende of it on the highest ioynt of the thumbe.

20 And for to expresse 20, you must set youre fingers straight, and the end of your thumbe to the partition of the formost and middle finger.

30 30 Is represented by the ioyning togither of the heads of the formost finger & the thumbe.

40 40 Is declared by setting of the thumbe crossewayes on the formost finger.

50 50 Is signified by right stretching forth of the fingers ioyntly, and applying of ye thumbes ende to the partition of the middle finger, and the ring finger or wedding finger.

60 60 Is fourmed by bending of the thumbe crooked, and crossing it with the forefinger.

70 70 Is expressed by the bowing of the fore-most finger, and setting the ende of the thumbe

betweene

betweene the 2 formost or highest ioyntes of it.

80 Is expressed by setting of the foremoste finger crosseswayes on the thumbe, so that 80 differeth thus from 40: for that 80, the forefinger is set crosse wayes on the thumbe, and for 40 the thumbe is set crosse ouer the forefinger. 80.

90 Is signified by bending the forefinger, and setting the end of it in the innermost ioynt of the thumbe, that is euen at the foote of it. 90.

And thus are all the numbers ended vnder 100.

Scholer. In deede these be all the numbers from 1 to 10, & then all the tenthes within 100, but this teacheth me not howe to expresse 11, 12, 13, &c. 21, 22, 23, &c. and such like. 11, 12, 13, 21,

Mayster. You can little vnderstand, if you can not doe that without teaching. What is 11? is it not 10 and 1? then expresse 10 as you were taught, and 1 also, that is 11: and for 12 expresse 10 and 2: for 23 set 20 and 3: and so for 68, you must make 60, and thereto 8: and so of all other sortes. 22, 23.

But now if you would represent 100, other any number aboue it, you must doe that with the right hande, after this maner.

You muste expresse 100 in the right hande with the little finger, so bowed as you did expresse 1 in the left hande. 100.

And

And as you expressed 2 in the left hande, the same fashion in the righte hande dothe de-
200 clare 200.
300 The fourme of 3 in the right hande standeth for 300.
400 The forme of 4 for 400.
500 Likewise the fourme of 5, for 500.
600 The fourme of 6, for 600. And to bee shorter: looke howe you did expresse single unities and tenthes in the left hande, so must you expresse unities and tenthes of hundredes, in the right hande.

Scholer. I understande you thus: that if
900 I woulde represent 900, I must so fourme the fingers of my right hande to expresse 9. And as in my left hande I expressed 10, so in my
1000 right hande must I expresse 1000.

And so the fourme of euerie tenth in the left hande, serueth to expresse like number of thou-
4000 sandes, so the summe of 40 standeth for 4000.
8000 The summe of 80, for 8000.

And the fourme of 90 (whiche is the grea-
9000 test) for 9000, and aboue that I cannot expresse any number.

Mayster. No, not with one finger, howe be it, with diuers fingers you maye expresse 9999. and all at one time, and that lacketh but

I of 10000. So that vnder ten thousande you may by your fingers expresse any summe. And this shall suffise for Numeration on the fingers. And as for Addition, Subtraction, Multiplication, and Diuision (which yet were neuer taught by any man as farre as I we know) I will instruct you after the treatise of Fractions: and now for this time farewell, and looke that you ceasse not to practise that you haue learned.

Scholer. Sir, with moste harty minde I thanke you, bothe for your good learning and also your good counsell, which (God willing) I truste for to follow.

FINIS.

THE SECONDE PART OF ARITHMETIKE, TOUCHING Fractions, brieflye sette foorthe.

SCHOLER.

Albee it I perceyue youre manifolde businesse doth so occuppye, or rather oppresse you, that you can not as yet completelye ende that treatise of Fractions Arithmeticall, whiche you haue prepared, wherein not onelye sundrye workes of Geometry, Musike, and Astronomy bee largely sette foorthe, but also diuers conclusions and naturall woorkes, touching mixtures of metalles, and compositions of medicines, with other straunge examples, yet in the meane season I cã not stay my earnest desire, but importunely craue of you some briefe preparation towarde the vse of Fractions, whereby at the leaste I maye bee able to vnderstande the common woorkes of them, and the vulgare vse of those rules, whiche without them can

not

not well bee wrought.

Mayster. If my leysure were as greate as my will is good, you should not neede to vse any importunate crauing for the attayning of that thing, whereby I maye bee perswaded that I shall anye wayes profite the common wealth, or helpe the honest studies of any good members in the same: wherefore, while mine attendaunce will permitte mee to walke and talke, I am well willing to helpe you as I maye.

Therefore, firste to beginne with explication of this name Fraction, what take you it to bee?

Scholer. Mary sir, I thinke a Fraction, (as I haue hearde it often named) to be a broken number, that is to saye, to bee no whole number, but a parte of a number. VVhat a Fraction is.

Mayster. A Fraction in deede is a broken number, and so consequently the parte of an other number: but that must be vnderstanded of suche an other number, as can not bee diuided into any other partes than Fractions: for although I may take the thirde parte of 60, or the fourth part of it, and so of other partes diuersly, yet these partes bee not proverly, nor ought not to bee called Fractions, bicause they

may bee expressed by whole nūbers: for yͤ third parte of it is 20: the fourthe parte is 15: the twelfth parte is 5, and so forth of other partes, which all be whole numbers.

Vhat a raction is roperly.

Wherefore properly a Fraction expresseth the partes or parte onely of an unite, that is to say, that the number which is the whole or entier summe of any Fraction, may not bee greater than one: and therefore it followeth, that no one fraction alone can be so greate, yͭ it shall make 1, as by examples I will declare as soone as I haue taught you to knowe the forme how a Fraction is expressed or represented in writing.

Numeration.

BUt first to beginne with the expressing of a Fraction, which is the numeration of it, you must understande that a Fraction is represented by 2 numbers, sette one ouer yͤ other, and a lyne drawen betweene them, as thus, $\frac{1}{3}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{10}{17}$, which foure Fractions you muste pronounce thus: $\frac{1}{3}$, one third parte: $\frac{3}{4}$, three quarters: $\frac{2}{5}$, two fifte partes: $\frac{10}{17}$, ten seuenteene partes.

Scholer.

Scholer. I vnderstande the forme of their expression and pronunciation, but their meaning or valuation seemeth more obscure: yet I thinke that by the two first Fractions I vnderstande the valuation of the two latter fractions, and so consequently of other.

Mayster. Value them then, that I maye perceyue your taking of them.

Scholer. $\frac{2}{5}$ betokeneth two fifte partes, that is to say, if one be diuided into 5 partes, that fraction doth expresse ij. of those fifthe partes: $\frac{10}{17}$ doth signifie, that if one bee diuided into xvij. partes, I muste take x. of them. And this I gather of the ij. first examples: for $\frac{1}{3}$, that is one thirde part, doth easily declare, that if any one thing bee diuided into iij. partes, I muste take but one of them: so $\frac{3}{4}$, that is iij. quarters, doth declare that one beeing diuided into iiij. quarters, I muste take (for this fraction) iij. of those quarters.

If there be no more difficulty in their Numeration, then I praye you goe forewarde to their Addition and Subtraction, and so to the other kindes of workes: for I vnderstand that the same kindes of workes bee in Fractions, that be in whole numbers.

Mayster. There are the same kinde of

workes in bothe, albeeit the order of them is diuers, as I will anon declare: but yet more in numeration before wee leaue it. You must vnderstande, that those two numbers whiche expresse a fraction, haue seuerall names. The ouermost which is aboue the line, is called the Numerator, and the other beneath the line, is called the Denominator.

Scholer. And what is the reason of their diuers names? For in mine opinion both bee Numerators, seeing both they doe expresse the numeration of the Fraction.

Numerator and Denominator.

Mayster. You are deceyued: for one onely (whiche is the ouermost) dothe expresse the numeration: and the Denominator doth declare the number of partes into whiche the vnit is diuided, as in this example, when I say: Diuide a pound weight of golde betwene iiij. men, so that the firste man shall haue $\frac{2}{15}$ the seconde $\frac{3}{15}$, the thirde $\frac{4}{15}$, and the fourth $\frac{6}{15}$.

Now do you perceiue ỹ by the Denominator (whiche is one in all foure fractions) it is intended, that the pounde waight shoulde bee diuided into so many partes, I meane 15, and by the iiij. seuerall numerators is limitted the diuers portion that eche man shall haue, that is, that when the whole is parted into 15, the first

man

man shall haue 2 of those 15 partes: the second man three of them: the third man 4: and the fourth man, 6. And so may you set the seuerall offices (as it were) of those two numbers, I meane of the Numerator and the Denominator.

And hereby you perceyue, that a man can haue no more partes of any thing than it was diuided into: nother yet aptly so many: so that it were vnaptly sayd: You shall haue $\frac{15}{15}$, that is xv, fifteene partes of anye thing, seeing it were better sayde: You shall haue the whole thing.

Scholer. So doth it appeare reasonablye: for the labour is vayne, to diuide anye thing, and then to apply the Diuision to no vse. And muche lesse reasonable were it to say $\frac{16}{15}$: for if the whole bee diuided into 15 partes onelye, it is not possible to take 16 of them, that is to say, more than altogither.

Mayster. This is true, touching the proper and apt vse of the name of a fraction: yet improperly and after a vulgare acceptation (for easinesse in worke) bothe those formes bee called Fractions, bicause they be written lyke Fractions, although they bee none in deede, for $\frac{15}{15}$, and generallye all suche other where the

Numerator & denominator bee equall, are not fractions: but the whole thing, with all his parts. And so $\frac{16}{12}$ is not to bee called a fraction, but a mixt number, of a whole number and a Fraction. For it is as muche, as $1\,\frac{4}{12}$, that is one whole one, and 4 twelue partes, as shall bee declared in Reduction. Therefore they do abuse the names, that call them fractions, where the Numerator is either equall or g[illegible]ater than the Denominator.

Scholer. But is there anye nee[illegible]ll cause why they should so abuse the name [illegible]

Mayster. There is cause why [illegible] shall sometimes, for easinesse in worke, [illegible]te some numbers after that sorte, like Fractions: but they needed not to call them fractions, but as they bee whole numbers, or mixt numbers (that is whole numbers with fractiōs) expressed like fractions.

Nowe muste you vnderstande, that as no fraction properly can bee greater than 1, so in smalnesse vnder one, the nature of Fractions doth extende infinitely: as the nature of whole numbers is to increase aboue one infinitelye, so that not onely one, maye bee diuided into infinite fractions or partes, but also euerye fraction may be diuided into infinite fractions

or partes, which commonly bee called Fracti- ons of Fractions, and they bee expreſſed di- uerſely: As for example $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$, that is three quarters of two third partes, of one halfe part. Whereby is ſignified, that if one bee diuided into two halues, and the one halfe into three partes, and two of thoſe three partes, bee diui- ded ioyntly into foure quarters, this Fraction of Fraction doth repreſent three of thoſe quar- ters.

Fractions of fractions.

Scholer. I pray you let mee proue by an example in common money, whether I doe rightly vnderſtande you or no. One Crowne, which I take for an vnit, dothe contayne 60 pennies, therefore the halfe of it is 30 pens: $\frac{2}{3}$ of that halfe is 20 pens, whereof $\frac{3}{4}$ is 15 pens, ſo then 15 pens is $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ of a crowne. And ſo 3 pens, is $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{3}$ of a ſhilling.

Mayſter. You perceyue this well inough, but howe happened that you founde no doubte in the forme of wryting theſe Fractions, ſeyng the two latter fractions haue no lyne betweene their numbers, as the firſte hath?

Scholer. Bycauſe I had forgotten (as ſcholers oft tymes doe) that that was tolde mee before: but I pray you, expreſſe the reaſon thereof.

Mayster. This forme is but voluntarye, and therefore hathe none other reason than the will of the diuiser, which forme many we folowe. Some other we make lines betweene euery fraction, and adde wordes of distinction, after this sorte, $\frac{3}{4}$ of $\frac{2}{3}$, of $\frac{1}{2}$, which forme is good also.

Some other expresse them thus in slope forme, to distincte them from seuerall Fractions of one whole number: for if they were set in one right line, thus, $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$, then ought it to be pronounced, three quarters, and two third partes and an halfe, which maketh almoste two whole vnits, lacking but one xij. parte. And so is it nothing agreable with the other Fraction of Fractions, wherefore it is a greate ouersight in certaine learned men, which doe expresse them so confusedly with such seuerall fractions, that a man can not knowe the one from the other.

$\frac{3}{4}$
$\frac{2}{3}$
$\frac{1}{2}$

Therefore some men (as Stifelius) doe expresse without a lyne numbers of proportion, being applyed to Addition or Subtraction: bicause they muste bee taken as two: where the line in fractions maketh them to bee taken for one: for of the Numeratour and Denominatour

tout is made one number.

Scholer. Then I perceyue there bee three seuerall varieties in fractions: First when one onely fraction is set for one number, as $\frac{4}{5}$, that is foure fifth partes. The seconde, is when there bee sette two or more seuerall fractions of one number, as $\frac{4}{9}\ \frac{2}{5}$, that is iiij. ninthe partes, and two fifth partes. The thirde sorte of Fractions of fractions, as $\frac{4}{9}\ \frac{2}{5}$, that is iiij. ninth partes of two fifth partes. Three seuerall varieties.

Maister. You haue sayde well, if you will vnderstande well your owne wordes.

Scholer. If it shall please you, I will by an example in the partes of an olde Englishe Angell expresse my meaning.

Maister. Let mee heare.

Scholer. The olde Englishe Angell did containe vij. ß. vj d. that is 90 d. Nowe $\frac{4}{5}$ of it, is 72 d. And of the same 90 d, if I take $\frac{4}{9}\ \frac{2}{5}$, that is foure ninthe partes, and two fifthe partes, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, whiche bothe make 76: but if I take $\frac{4}{9}\ \frac{2}{5}$, that is foure ninthe partes of two fifte partes, seeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yelde but 16: for $\frac{1}{9}$ of 36 is but 4, and that taken foure tymes, maketh 16.

Maysters. This is plainely expressed and

truelye, and hereby (I doubt not) but you we perceyue, that as greate a difference as is betweene 16 and 76, so muche difference is betweene these two fractions $\frac{4}{9}\frac{2}{3}$, and $\frac{4}{9}\frac{2}{5}$.

The order of vvorke in fractions

And now that you vnderstande these varieties, I will proceede to the rest of the workes: firste admonishing you that there is an other order to bee followed in Fractions than there was in whole numbers, for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diuision, and Reduction, but in fractions to followe the same aptnes in proceeding from the easiest workes to the harder) wee must vse this order of the woorkes, Numeration, Multiplication, Diuision, Reduction, Addition, and Subtraction.

Scholer. That Multiplication and Diuision shoulde go togither, and Subtraction to follow Addition, naturall order dothe perswade: but why Multiplication should be first in order heere nexte to Numeration, and Reduction in the middle, I desire to vnderstande the reason.

Maister. As in the arte of whole numbers order woulde reasonably beginne with the easiest, and so go forewarde by degrees to the hardest,

hardest, euen so reason teacheth in Fractions the like order. And considering that Addition or Subtraction of Fractions can very seldome be wrought without multiplication and Reduction: and contrary wayes Multiplication and Reduction may bee wrought without this forme of Addition or Subtraction, therefore was it orderly required, that Multiplication and Reduction shoulde goe before Addition and Subtraction. And the same reason serueth for the placing of Multiplication before Reduction.

Scholer. Then, if Multiplication bee the easiest, I pray you declare the forme of it, firste by rule, and then by example.

Mayster. Your request is good.

Multiplication.

Therefore when any two Fractions be proponed to be multiplyed togither, the Numerator of the one must be multiplyed by the Numerator of the other: and the summe that amounteth thereof, must be set for a newe numerator: likewayes the Denominator of the one must bee multiplyed by the denominator of

the other and that that amounteth shall bee set for the common denominator: and this newe thirde Fraction expresseth the producte of the multiplication of the two first Fractions proponed, whereof take this example, $\frac{3}{5}$ multiplyed by $\frac{5}{12}$, doth make $\frac{15}{60}$.

Scholer. I perceyue then, that 3 beeing the Numeratour of the firste Fraction, is multiplyed by 5, beeing the Numeratour of the seconde Fraction, whereof amounteth 15, the Numerator of the thirde fraction. And so likewayes, 5 beeing denominator of the first Fraction, is multiplyed by 12 the Denominator of the seconde fraction, whereof amounteth 60 the new denominator: so that I perceyue how the worke is done, but I doe not perceyue how $\frac{15}{60}$ is greater than $\frac{3}{5}$: For if I shall vse my former maner of examination by the partes of some coyne, I see that $\frac{3}{5}$ of a crowne, is 36 d, and $\frac{5}{12}$ of a crowne, is 25 d, whereof the one multiplied by the other, dothe make 900 d, whiche is 15 crownes: but by youre multiplication there amounteth $\frac{15}{60}$, whiche is but 15 d, and that is muche lesser than any of bothe the first fractions.

Mayster. That difference is betwene multiplication in whole numbers, and multiplication

cation in broken numbers,that in whole numbers the summe that amounteth , is greater than both the other wherof it came : but in fractions it is contrary wayes : for the summe that amounteth is lesser than anye of the other two fractions,whereof it came.

Scholer. I desire muche to vnderstande the reason thereof.

Mayster. Although I purposed to reserue the reasones of workes Arithmeticall for the perfect booke of Arithmetike, yet I will shewe you this, bicause of the straungenesse of the worke.

You see in whole numbres, that of two numbres beeing multiplyed together, is made the thirde number : which thirde number doth beare the same proportion to the number multiplyed,that the multiplyer doth beare to an vnit. And so in Fractions, the thirde number whiche amounteth of multiplication , beareth the same proportion to eche of the two firste fractions, that the other of those two fractions doth beare to an vnit.

Scholer. Sir, I vnderstande your wordes thus : When 40 is multiplyed by 12, there doth amounte 480, whiche 480 doth contayne 40 so many times in it , as 12 dothe

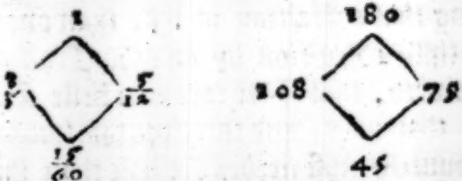
containe

conteyne 40 ſo many times in it, as 12 dothe conteyne vnits: that is to ſay, twelue times. And ſo it appeareth, that 480 doth containe twelue ſo many times alſo, as 40 doth contayne Units, that is 40 times. But nowe I ſee not how the thirde number in this example of Fractions can conteyne any of the two former (as it happened in whole numbers) ſeeing it is leſſer than eyther of them.

Mayſter. No meruayle, if you cannot ſee that thing whiche is not poſſible to bee ſeene of any man, howe the thirde number in multiplication of Fractions ſhould bee greater than any of the two former Fractions, but yet this maye you ſee (whiche I ſayde) that the thirde number in Fractions ſo multiplyed, dothe beare the ſame proportion to anye of the two former Fractions, that ẏ other of thoſe ij. Fractions doth beare to an Unit, as in youre example $\frac{3}{5}$ being multiplyed by $\frac{5}{12}$, dothe make $\frac{15}{60}$. Now ſay I, that $\frac{15}{60}$ dothe beare the ſame proportion to $\frac{3}{5}$, that $\frac{5}{12}$ dothe beare to an vnit, as you maye in your owne forme of examination by coyne trye it. For in an olde Angell are 180 halfe pens, whiche I ſet for the entire vnit, whoſe partes (according to ẏ Fractions aforeſayde) are theſe: for $\frac{15}{60}$ ſette 45 ob.

for

for $\frac{3}{5}$ take 108 ob. and for $\frac{5}{12}$ put 75 ob. Nowe dothe 45 beare the same proportion to 108 that 75 dothe beare to 180: for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180. And for easier applying of eche comparison, consider this forme of setting all these numbers before your eyes.



But these reasons may bee better reserued till an other time, when the knowledge of proportions in due order shall bee taught. Yet in the meane season I will shewe you howe it commeth to passe, that in Fractions the thirde summe muste needes bee lesser than any of the other two.

Consider thus, that when a Fraction is proponed, as in the former example $\frac{3}{5}$, if it bee multiplyed by more thā one, it will make more than one entier nūber. As if I multiply this $\frac{3}{5}$ by 5, that is to say, if I take it fiue tymes, it will make three entiere vnits: example in a crowne, $\frac{3}{5}$ of it maketh 3 ß, which if I take

fiue

fiue times, it will amounte to 15 shillings, that is three entier crownes: so if I take the same $\frac{3}{5}$ but twice, it will yelde 6 ß, that is one entier crowne and $\frac{1}{5}$. Nowe if I take it but ones, it cannot bee more than it was before, that is 3 ß. And if I take it lesse thā once, it can not bee so much as it was before. Then seeing that a Fraction is lesse than one, if I multiply a Fraction by an other Fraction, it followeth, that I doe take that firste Fraction lesse than once, and therefore the summe that amounteth must needes bee lesse than the firste Fraction.

Scholer. Sir, I thanke you much for this reason: And I truste, I doe perceyue the thing, as by example of this same fraction $\frac{3}{5}$ I will expresse. If I take $\frac{3}{5}$ of a crowne once, that is to saye, if I multiply $\frac{3}{5}$ by 1, it will bee as it was before, but 3 ß, so if I doe multiply it by $\frac{1}{2}$, that is, if I take it but halfe one time, then will it bee but halfe so much: likewaies if I multiply it by $\frac{1}{3}$, that is, if I take but the thirde parte of once, it will yelde but 12 pens, that is the thirde parte of the firste Fraction.

And so to make an ende. If I take it but the twelfthe parte of ones, that is, if I doe multiply

tiply it by $\frac{1}{12}$, it will yelde but the twelfthe parte of the firste Fraction, which is but 5 pens. And it followeth, that if $\frac{1}{12}$ make 3 pens, then $\frac{5}{12}$ muste needes make fiue tymes so much, that is 15 pens, which was the summe that hath giuen the occasion of all this doubte.

Mayster. Then I perceyue you haue sufficient vnderstanding in this sort of Multiplication for this tyme, wherefore I will omitte that I might saye more of Multiplication, till wee come to Reduction, and will passe to the other workes, and firste to Diuision, whose place followeth Multiplication, bothe by naturall order, and also in easinesse of worke.

DIVISION.

When so euer two Fractions bee proponed, that the one shoulde bee diuided by the other, I muste sette downe first the Fraction that shall bee diuided (which is called the Diuidend) and then after it the other, which is the Diuisor. Then

 shall

ſhall I multiply the numerator of the diuidend by the denominator of the diuiſour, and that whiche amounteth, I muſt put for a newe numerator. Agayne, I ſhall multiply the denominator of the diuidend by the numerator of the diuiſor, and the number that amounteth thereof, I muſt put for the new denominator. And this thirde fraction is the quotient of the ſaide diuiſion.

Scholer. This ſeemeth eaſie in forme, as by example, thus: If I woulde diuide $\frac{5}{8}$ by $\frac{2}{6}$, firſte I muſt multiply 5 (being the numerator of the diuidend) by 6, whiche is the denominator of the diuiſor, and thereof riſeth 30: then I multiply 8 (beeing the denominator of the diuidende) by 2, beeing numerator in the diuiſor, and ſo riſeth 16, the whiche I muſte make in a thirde fraction, thus, $\frac{30}{16}$.

Mayſter. Mee ſeemeth you are quicker in vnderſtanding nowe, than you were when I taught you the arte of whole numbers: but that is no maruaile, for the more knowledge that any man getteth, the readyer ſhall he finde his witte, & quicker in vnderſtanding: but yet of ij. things I will admoniſhe you, whiche you myght haue obſerued heere for eaſe of worke and lyghtnes of vnderſtanding the nature of the

the Quotient.

Whensoeuer you do diuide one Fraction by an other, either they be both equall togither, either els ẏ one is greater than the other: if they be equall, their quotient shall be suche, that the numerator and the Denominator of it shall be equall also. And if the two first Fractions bee vnequall, their quotient shall declare the same by the vnequalitie of the numeratour and denominatour, as in these examples following shall appeare.

First of equall Fractions: $\frac{4}{9}$ and $\frac{12}{27}$ bee equall togither: and if the one bee diuided by the other, the quotient will bee $\frac{108}{108}$, as you may perceyue by that rule aforesayd.

Now in the vnequall fractions, as $\frac{8}{9}$ and $\frac{3}{10}$ the quotient will bee $\frac{80}{27}$: where the numerator is greater than the denominator.

Scholer. I see it is so, but I see not the reason why it should be so.

Mayster. The reason is this. When any fraction is diuided by an other, the quotiente declareth what proportion the diuidende beareth to the diuisor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$, maketh 2, whiche muste bee sounded, not two, but twice: declaring that $\frac{1}{4}$, is contayned twice in $\frac{1}{2}$.

Note how to know the proportion betweene numbers.

And note this, that the Numerator in the Quotient, representeth the Diuidend, and the Denominator representeth the Diuisor. And this is alwayes true, whether the greater fraction be diuided by the lesser, or the lesser by the greater. But this proportion will not bee exactly knowne, till you haue learned the arte of Proportions: notwithstanding somewhat of it will I declare in the next rule of Reduction. But nowe for the easie remembrance of the Quotient in Diuision, assone as you haue set downe your two fractions, the one againste the other, then make a streight line for the quotient: and as soone as you haue multiplyed the Numerator of the diuidend, by the Denominator of the diuisor, set the number that amounteth, ouer the sayde line, and then multiply the other two numbers, and set theyr totall vnder the same line.

Scholer. I perceyue you would not haue me trust to memorie till I were better experte, leaste oftentimes I happen by misse remembraunce to bee abused. This example I take for that declaration.

If I would diuide $\frac{2}{3}$ by $\frac{3}{4}$, I must set the numbers one agaynst the other, (as here doth appeare) & then make

$$\frac{2}{3} \quad \frac{3}{4}$$

an other line for the Quotient in some good distance, where I maye sette the numbers of the Quotient, as soone as any of them is multiplyed: So then as soone as I haue multiplied 2 by 4, which maketh 8, I shall set that 8 ouer that line, thus. $\frac{8}{\ }$ and then multiply 3 by 3, whiche yeldeth 9: and that 9 must I set vnder the same line: and then will the whole Quotient appeare thus $\frac{8}{9}$. Wherby appeareth (as I remember your wordes) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9: but how may I perceyue that? Note.

Mayster. Althoughe you shall better perceyue it by the rule of Reduction, yet this example may bee declared in common coyne, as in a common shilling of xij. pens, of whiche $\frac{2}{3}$ maketh 8 d, and $\frac{3}{4}$ dothe make 9 pens, and so you may easilye see that theyr proportions doe agree. And if you had taken this example before, when you tooke the example of $\frac{5}{8}$ and $\frac{2}{6}$, your Quotient would appeare (as this doth) more easier to vnderstande, where as that quotient beeing $\frac{30}{16}$, is not an easie proportion for you to perceyue, beeing yet little acquainted with proportions: whereof to giue you some taste, I will enter to the rule of Reduction: in whiche also I will declare other workes,

both of Multiplication and also of Diuision, which now I must for a time omit, as things that do neede the helpe of reduction.

REDVCTION.

iue varieties of Reduction.

Therefore will I now declare the diuersities of Reduction of Fractions, whiche commonly haue fiue varieties.

1 First, when there be sundrie Fractions of one entire Unit, they must bee reduced to one denomination, and also into one Fraction.

2 Secondarily, when there bee proponed fractions of Fractions, they must bee reduced likewayes into one fraction, for otherwayes they cannot be brought into one denomination.

3 Thirdly, when an Improper Fraction is proponed, that is to saye, a fraction in forme, which in dede is greater than an Unit, it must be reduced into apte forme, expressing the Unit or Units of it, and the proper fraction distinctlye. And sometimes also it shall bee needfull to conuerte suche a mixte number of Units, with Fractions into the forme of a Fraction, that is into an Improper Fraction, whiche 2

formes

formes I esteeme but as one, bicause they worke on one kinde of number.

4. Fourthly, there happeneth sometimes fractions to be written in greate numbers, which myght bee written in lesser numbers, therefore is there a meane to reduce such greate numbers into their smallest tearmes.

5. Fiftly, when any Fraction betokeneth the partes of a whole thing, which hath by common partition certaine partes, but none of like denomination with that fraction, then maye you reduce the sayde Fraction into an other, whose Denomination shall expresse the common partes of that whole thing.

Scholer. This distinction in doctrine delyghteth mee much, but more with hope than present frute, for as yet I doe not understande scarsely the varietyes, and much lesse the practise and use of their workes.

Mayster. Reduction is an orderly alteration of numbers out of one forme into an other, which is never done orderly but for some needefull use, as in every of the sayde 5 severall varieties I will distinctly declare.

Firste therfore, when two or more severall Fractions of any Unite bee proponed, as for example, $\frac{1}{16}$ and $\frac{4}{6}$: bicause it is harde to tell what — The firste sorte of Reduction.

 what

what portion of the entier number those two Fractions we expresse, therefore was Reduction diuised, to bee a meane whereby these seuerall Fractions might be brought into one Denomination and Fraction.

And in these Fractions this is the arte for bringing them to one denomination.

Hovve to reduce Fractions of diuers Denominations into one Denomination.

Multiply firste the Denominators togither, & the totall thereof you shall set twice downe vnder two seuerall lines for two newe Denominatours, or rather for one common Denominatour: Then multiply the Numerator of the firste Fraction, by the Denominator of the seconde, and set the totall thereof for the Numerator ouer the firste lyne. Likewise multiply the Numerator of the seconde Fraction by the Denominator of the firste, and set that totall ouer the seconde lyne for the Numerator of that Fraction, and so are those two firste Fractions of seuerall denominations, brought to one Denomination.

Scholer. If I vnderstand you, as I thinke I doe, my example shall declare the same. The Fractions which you propwned were these, $\frac{3}{16}$, and $\frac{5}{6}$, whose Denominators (beeing 16 and 6) I multiplye togither, and there amounteth 96, which I set vnder ij. lynes, thus. $\frac{\ }{96}$ $\frac{\ }{96}$

Then

Then I multiplye the Numerator of the firste fraction by the Denominator of the seconde, saying: 3 into 6 maketh 18, that set I ouer the firste lyne for a newe Numeratour, and it will bee thus. $\frac{18}{96}$

Likewayes I multiply the Numeratour of the second Fraction by the Denominatour of the firste, saying: 4 times 16 maketh 64, that I sette for the seconde Numerator, and the Fraction will appeare thus. $\frac{64}{96}$

so that bothe Fractions brought to one Denomination, muste stande thus: $\frac{18}{96}$ and $\frac{64}{96}$

Mayster. You haue done well.

Scholer. I beseeke you, let me examine it after my accustomed forme, by common partes of coyne.

Maister. Go to.

Scholer. A new Angell accompted at eight shillings, contayneth 96 pens, whereof $\frac{1}{16}$ that is the xvj. parte, is sixe pens, and $\frac{3}{16}$ is 18 pens, that is $\frac{18}{96}$. Agayne $\frac{1}{6}$ of the same Angell, is 16 pens, so that $\frac{4}{6}$ maketh 64 d, that is $\frac{64}{96}$. And so I finde the summes to agree with the other before.

 Mayster.

Note the Reduction of three Fractions (or more) to one.

Mayster. So haue you nowe the arte to bring such two Fractions into one Denomination. And if there be more than ij, then must you multiply all the Denominators togyther, and set the totall thereof so many times downe as there bee Fractions, and then to get for eche one a newe Numerator. Multiply the Numerator of the first, by the Denominator of the seconde, and the totall thereof multiply by the denominator of the thirde, and so forth if there be more. Likewise multiplye the Numerator of the seconde, by the Denominator of the firste, and the totall thereof by the Denominator of the thirde. And in the same sorte multiply the Numerator of the thirde into the Denominator of the firste: & the totall thereof into the Denominator of the seconde, and so forthe, if there were mo. So these 3 Fractions $\frac{2}{5}$ $\frac{3}{4}$ $\frac{1}{3}$ dothe make by Reduction these other 3 Fractions of one Denomination $\frac{24}{60}$ $\frac{45}{60}$ $\frac{31}{60}$. All which you maye bring into one Fraction by adding the Numerators togither, and putting that totall for the common Numerator, reseruing still that same common Denominator. And those 3 Fractions make one Improper Fraction, thus. $\frac{100}{60}$

Scholer. All this I perceyue, and also that

this

this last Fraction is more than an Unit, and therefore you did cal it an Improper Fraction.

Mayster. There bee certaine other formes of working in this reduction, whiche I will briefly touche also, to giue you an occasion to exercise your wit therein.

The first varietie of this Reduction.

The firste varietie is this. When you haue made and written downe your common Denominator (as I haue taught before) then to get a Numerator for the first, doe thus. Diuide the common Denominator by the Denominator of the first Fraction, and the quotient multiplyed by the Numerator of the same, yeldeth a newe Numerator for the firste newe Fraction. So likewise doe with the seconde and the thirde, and with all the residue if there be more.

Scholer. That will I proue in youre laste example of these 3 Fractions $\frac{2}{5}\,\frac{3}{4}\,\frac{2}{3}$. When the Denominatours be multiplyed, they make 60, for 5 into 4 maketh 20, and 20 by 3, yeldeth 60, that I sette downe 3 times, thus. $\overline{60}\ \overline{60}\ \overline{60}$: then to haue a Numerator for the firste, I muste diuide 60 by 5, (the Denominator of the first) and the quotient is 12, whiche I must multiply by 2 (the Numerator of the first) and that maketh 24, and so haue I for the first Fraction $\frac{24}{60}$.

Likewise

The second varietie. Likewise for the seconde fraction: I diuide 60 by 4, and there commeth 15, whiche I multiplie by 3, and so haue I 45 and the seconde fraction $\frac{45}{60}$. Then for the thirde in like sort will come $\frac{40}{60}$.

Mayster. An other way is this. If it happen so that the lesser Denominator can by any multiplication make the greater, then note the multiplier, and by it multiplie the Numeratour ouer that lesser Denominatour, and for the lesser Denominatour put the greater, as thus in these two Fractions, $\frac{1}{12}$ $\frac{2}{3}$, three being the lesser Denominatour multiplyed by 4, will make 12, whiche is the greater Denominatour: therefore by the same 4, I doe multiply 2, whiche is Numerator ouer 3, and that maketh 8: vnder whiche I put 12 beeing the greater Denominatour, whiche is also made by Multiplication of 4 into 8, & so haue I these ij. Fractions $\frac{1}{12}$, $\frac{8}{12}$: thus shortlye reduced without altering the one Fraction.

Scholer. This I vnderstande.

The thirde varietie. Mayster. Then marke this thirde waye: If the denominators doe not happen so, that one by multiplication maye make the other, then looke whether they bothe maye bee partes

of any other one number, as in $\frac{5}{12}$ and $\frac{7}{18}$, although the lesser taken but twise bee to greate to make 18, yet they bothe may bee partes vnto 36: therefore looke howe many times 12 is in 36, and that quotient beeing multiplyed by the numeratour ouer 12, the totall shall bee put in steade of the Numeratour ouer 12, and for 12 put 36, thus, $\frac{15}{36}$. So lykewyse looke how often is 18 in 36, and bicause it is twice, therefore by 2 multiply 7, whiche is ouer 18, and it will bee 14, set that for the numeratour, and in steade of 18 put 36, and then shall your fractions reduced, stande thus, $\frac{15}{36}$ $\frac{14}{36}$ in steade of $\frac{5}{12}$ and $\frac{7}{18}$.

And if you will proue whether you haue wrought well or no, that maye bee proued by Reduction of them againe to their former denominations, whiche arte shall bee taught in the fourth kinde of Reduction, where greater termes of Fractions be reduced into smaller in nūber, but no smaller in proportion. And if in suche Reduction the same termes or numbers come againe that were before, then is the work good, else not. Proofe.

Scholer. Sir, I heare your wordes, but I doe not vnderstande many of them, whiche it may please you to declare.

Mayster.

Mayster. With a good will, when conuenient place serueth, but that muste bee in the sayd iiij. kinde of Reduction: In the meane season I will declare the second forme of Reduction, whiche teacheth howe to reduce Fractions of Fractions into one Fraction, and so to one Denomination.

Reduction of fractions of fractions into one fraction and denomination.

When fractions of fractions bee proponed, you shall multiply the Numeratoꝛs of eche into other, and set the totall for the newe Numeratour, then multiplie all the Denominatours, lykewayes, and take their totall for the newe Denominatoꝛ, and so are they speedilye reduced.

Scholer. If that be all, then I vnderstand it alreadie, as by this example I will declare. These bee the Fractions, $\frac{3}{4}\,\frac{2}{3}\,\frac{6}{7}\,\frac{7}{9}$, whiche I would reduce to one Denomination.

Therefore begin I with the Numeratours, and multiplie them all togither, saying: 3 into 2 maketh 6, and 6 by 6 maketh 36, whiche multiplyed by 7, yeldeth 252, that I set ouer a line for the Numeratour, thus: $\frac{252}{\quad}$

Then I multiply the denominatours, 4 by 3 maketh 12, and that by 7 bringeth 84, which multiplyed by 9, yeldeth 756, the new Denominatour.

minatour. And so the whole reduced fraction is this, which is

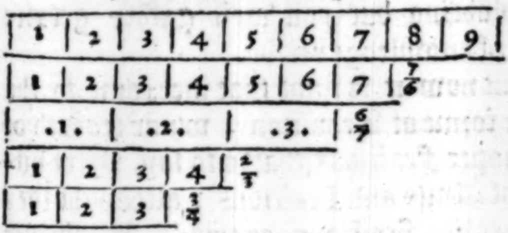
$$\frac{252}{756}$$

to harde a Fraction for mee to understand yet.

Mayster. You thinke so, and no maruaile, but anone you shall learne to iudge it easilye, for this Fraction is no more in deede then $\frac{1}{3}$, although it bee in greater tearmes, and therefore more straunger and more obscure.

And this suffiseth for this Reduction, saue that I will shewe you by a figure of measure, the iuste rate and reason of this kinde of Fractions, and also the due understanding of the Reduction.

The entier measure parted into 9.



Heere you see the longest measure, (which standeth for the whole and entier quantitie) first parted into 9 Diuisions, whereof 7 are seuered by the seconde measure: and thereof agayne

gayne are parted out 6. And that 6 beying distincte into 3 partes, 2 of them are parted by the fourthe measure, of which fourth measure, beyng diuided into 4 partes, the lowest measure dothe contayne $\frac{3}{4}$, so that the same $\frac{3}{4}$ muste be named, not $\frac{3}{4}$ of the whole measure, but in deede is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$: or as I woulde rather expresse it, $\frac{3}{4}\,\frac{2}{3}\,\frac{6}{7}\,\frac{7}{9}$.

Scholer. This example is so sensible, that I can not chose but see it. And furthermore, I see also, that the same fraction is equall to $\frac{3}{9}$ of the entier measure, as the lynes which runne vp and downe we expressely set forthe. Also I see heere, that $\frac{2}{3}\,\frac{6}{7}\,\frac{7}{9}$ is equall to $\frac{4}{9}$. And further yet, that $\frac{6}{7}\,\frac{7}{9}$ is equall to $\frac{6}{9}$.

Mayster. I am glad that you see it so well, not doubting but you wyll gather greater lyght of knowledge hereby.

The thirde forme of Reduction.

But now it is tyme that wee come to the thirde forme of Reduction, which teacheth of Improper Fractions, that is to say, Mixt nūbers of Units and Fractions, althoughe they appeare like Fractions, as this $\frac{26}{5}$, which doth include 5 Units wholly, and $\frac{1}{5}$ ouer. Wherefore first you shall knowe them, by that the Numeratour is greater than the Denominatour.

Scholer.

Scholer. In deede sir that appeareth reasonable, that if the Numeratour doe expresse more partes to be taken of any Unite, than the Denominator dothe signifie that Unit to be diuided into, it must needes follow, that such a Fraction importeth more thā the whole, that is to say, the whole with certayne partes ouer. But what Reduction is there in it?

Mayster. There bee two seuerall kindes of Reduction, cōcerning such Fractions. Sometymes it shall be needefull to conuert those fractions into the Unites and the proper fraction that will remayne. And sometymes contrary waies, it shall be mete to reduce mixt numbers, that is Units, writtē with fractions into the forme of one simple Fraction, and so bee there two wayes.

Scholer. What is the meane of the firste way to turne improper Fractions into Units with their proper Fractions?

Mayster. That is thus. Your Numerator being greater than the Denominator, must be diuided by the same denominator, & the Quotient thereof expresseth the Units, the remayner shall be put for the Numerator of the fraction that resteth, and the denominator must be the same that was before.

Reduction of improper fractiōs into vnits vvith their proper Fractions.

Scholer. For example, I take $\frac{17}{5}$. And diuiding 17 by 5, the quotient will bee 3, and there will remayne 2.

Mayster. That muste you write thus, $3\frac{2}{5}$, where (you see) I haue written 3 without anye lyne, as entier numbers ought to be written, and the 2 that remayned, I haue set ouer the former denominator with a lyne, as a Proper fraction. And this number doth signifie now 3 Unites, and $\frac{2}{5}$ of one.

Scholer. Then if I would by Unites here vnderstand crownes, so it were 3 crownes, and $\frac{2}{5}$, that is 2 ß.

Mayster. Euen so, and therefore $\frac{17}{5}$ did signifie the same. But this happeneth sometimes, that whē the Reduction is so wrought, there remayneth nothing. And then is it not a mixt number, but a simple entier number, represented like a Fraction.

Scholer. As $\frac{15}{5}$ will make 3 iuste, and $\frac{18}{3}$ will make euen 6. This I will remember. But nowe, what is the seconde forme of Reduction, that you spake of for these sortes of Fractions?

Mayster. Whensoeuer you haue enye of these two sortes of numbers, that is to saye, whole numbers without fractions, or whole numbers

Reduction of whole numbers,

numbers with fractions, and you would turne them into the forme of a Fraction, you muste multiply the whole number by that denominatour, whiche you will haue to remaine still, and to the totall thereof adde the numeratour which you haue alreadye, and all that shall you set for the newe numerator, keeping still the former denominatour: as if you haue $6\frac{3}{4}$, which you woulde conuert into an Improper fraction, you must multiplie 6 by 4, whereof commeth 24, and thereto adde the numeratour whiche is 3, and so haue you 27 for the Numeratour, and 4 still for the Denominatour. either alone, or ioyned vvith fractions into fractions.

Scholer. Then is $\frac{27}{4}$ equall to $6\frac{3}{4}$.

Mayster. Euen iuste, and so backewarde (as appeareth by the former Reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of these Reductions may be the proofe of the others worke.

Scholer. This I perceyue: but nowe if you would turne whole numbers without fractions into any fraction, I see not howe that may be done, bicause there is no denominatour to make the Multiplication by.

Mayster. That was well marked: but this you knowe, þ no man intendeth to turne any whole number into a fraction, but he hath

in his minde that denominatour by which the Multiplication muste bee made: for the profe whereof I set downe 7, whiche is a whole number. And if you will haue this number conuerted into any certaine Fraction, will me to do it.

Scholer. I praye you reduce 7 into a Fraction.

Mayster. Then you care not what the fraction be, so it be some fraction.

Scholer. No, I passe not for the sort of the Fraction.

Mayster. Then howe can you thinke that you require mee to doe anye thing certaine, when you leaue me to doe as I list? And seeing you stande at that staye, whether thinke you that I must firste intende in minde what fraction I will make of it, before I can doe it in deede?

Scholer. Else you should do ignorantly.

Mayster. Then I will limit my selfe (seeyng you will not) to turne it into quarters. And therefore I multiply 7 by 4 (which is the denomination of quarters) and there amounteth 28 to bee set for the Numeratour, and the 4 must bee set for the Denominatour, and the Fraction will be thus. $\frac{28}{4}$

Scholer,

Scholer. In deede I perceyue this to bee reasonable, for without much tryall I understand that $\frac{28}{4}$ of any thing, doeth make 7. And so then if I would turne 8 into fifte partes, it will make $\frac{40}{5}$, whiche is all one with eighte. for 8 crownes turned into fifte partes, (that is into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a crowne.

Mayster. Seeing you understande nowe these three kinde of Reduction, I will declare unto you the fourth kynd, that is when Fractions be written in greater tearmes than they neede, howe they maye bee brought to lesser Tearmes.

Scholer. To write any Fraction in greater Tearmes than needeth, seemeth to bee a faulte, and so this rule seemeth to amende that fault. The fourth kinde of Reduction.

Mayster. It were a fault to do any thing without neede, which after must bee redressed: but in this case it is not so: neyther did I say absolutely (as you doe) that it needeth not to expresse those Fractions in so great Tearmes, but that the Fractions doe not neede, I meane for their value to bee understanded: but yet it maye bee needefull for the ease of those workes whereto they be applyed, as for example: In

 the

the firste kinde of Reduction this was youre owne example: $\frac{3}{16}$ and $\frac{4}{6}$, whiche when you would reduce, you were fayne to turne them first into one Denomination, and so appeared they thus. $\frac{18}{96}$ and $\frac{64}{96}$, where the Fractions (for their owne understanding) neded not to be turned out of smaller termes into greater, but yet the easinesse of working needed it.

Scholer. Sir, I understande nowe, not onely the difference of this neede (for the Fractions might better bee understanded as Fractions seuerall, eche in his value, when they were in lesser Tearmes, although they coulde not so well be reduced) but also I understand what you meane by greater Tearmes and lesser Tearmes, whereof before I was in doubt, for I see you call the Numeratour and Denominatour, the Termes of the Fraction.

Tearmes of fractions.

Mayster. I am glad you understand it so well. Nowe then when you would value any Fractions (bicause that may best bee done when the Tearmes are smallest) you shall reduce them to the smallest that you can, whiche thing you may do thus: Diuide the greatest of any suche two Tearmes by the lesser, and if any thing remaine by that remainer, diuide the last Diuisour: and if any thing remayne

Reduction of fractions into their smallest tearmes.

now,

now, by that, diuide the laste Diuisour (which was before the remayner of the first Diuision) and so continue still, till nothing be remaine in the Diuision: and then marke youre laste Diuisour, for it is the number that will easilye reduce youre Fraction, if you diuide bothe the Numeratour and the Denominatour by the same number, and put for the Numeratour the quotient of his Diuision, and for the Denominatour also his quotient, that riseth by his Diuision.

Scholer. I take for example $\frac{18}{96}$, and bycause 96 is the greater number, I diuide it by 18, and the quotient is 5, and there resteth 6: what shall I be with this quotient?

Mayster. Nothing in this worke, but now seing there remaineth somewhat, by that remayner must you diuide the laste diuisor.

Scholer. If I shall diuide 18 (which was the laste diuisour) by 6, that was the remayner, so is the quotient 3, and nothing resteth.

Mayster. As for the quotient, I omitte him yet: but bicause there dothe remayne nothing, therefore is 6 (which was youre laste diuisor) that number by which you maye reduce the fraction proponed.

Scholer. Then as you taught me, I must

diuide the numerator 18 by 6, and the quotient is 3, which I must put for the numerator ouer a lyne, thus. $\frac{3}{\ }$ and then by the sayde 6, muste I diuide also the denominator 96, and the quotient will bee 16, which I muste take for the denominator, and so is the fraction $\frac{3}{16}$. And so me thinketh this rule doth proue the worke of the firste Reduction.

Maister. That is true, if the firste Reduction were made of Fractions in their leaste tearmes, and els not, without some helpe, as the seconde number in that place will declare.

Scholer. The second number was $\frac{4}{6}$, which was turned into $\frac{46}{96}$, by that rule. Nowe yf I shall by this rule reduce it agayne into the leaste tearmes, I muste diuide 96 by 64, and there resteth 32, by which 32 I diuide 64, and there remayneth nothing: wherefore I muste take that 32 for the diuisour, to reduce the sayde fraction. Then we I diuide 64 by 32, and the quotient is 2, which I set for my Numerator. Agayne, I diuide 96 by 32, and the Quotient will bee 3, and so haue I but $\frac{2}{3}$.

Maystter. Muse not at the matter, for you haue done well ynoughe: but you thinke you

you haue not the fraction that you looked for, that is $\frac{4}{6}$, yet haue you one equall to it, as by the partes of a shilling you may prooue.

Scholer. Truth it is, for eche of them will bring forth 8 pens, so that, $\frac{8}{12}$ and $\frac{4}{6}$ & $\frac{2}{3}$, bee all three equall. And now I perceyue, that bicause $\frac{4}{6}$ was not written in the leaste tearmes that it myght bee, therefore this Reduction brought forth not it, but that other which is written in the leaste tearmes. Now vnderstande I this rule well. But is there any other way to worke this Reduction?

Maister. Yes, but first note this, that if you finde no such Diuisor to reduce the Fraction tyll you come to 1, bycause 1 dothe make no Diuision, therefore that Fraction is already in his leaste Tearmes, as by $\frac{71}{100}$ you may proue, and so of $\frac{85}{98}$, and many other like. But nowe to your question, if you can without that Diuision by memorye espye the greatest number that maye diuide exactlye bothe Tearmes of your Fraction proponed, then neede you not to vse that Diuision, as in this Fraction $\frac{60}{96}$, I see that 12, is the greatest number that can diuide them bothe: and therefore without anye worke, by memory onely, I turne that into $\frac{5}{8}$, but this abilitye in knowledge is gotten by

An other vvay for to vvorke Reduction

 exercise.

exercise.

Another way.

Yet one other waye of easie Reduction in this kinde there is, when your Fraction hathe any Cyphers in ye first places of both tearmes, then may you by casting awaye the Ciphers, make a briefe Reduction, as thus $\frac{300}{400}$, here take away the Cipher, and it will be $\frac{3}{4}$, which is the same in valew with $\frac{300}{400}$.

Scholer. And so if I haue $\frac{400}{650}$ it will be $\frac{40}{65}$.

Mayster. You are deceyued, for you take away more ciphers from the numerator, than you doe take from the denominator, which you may not doe.

Scholer. I confesse my faulte, which came of to much haste, I was more gladder of the rule than wise in vsing it: but nowe I vnderstand it, I trust.

The fifte kinde of Reduction.

Mayster. Then may I go in hande with the fift or laste kinde of Reduction, which teacheth how to turne any fraction proponed into any other denomination that you liste: or into any partes of common coyne, waightes, or measures, or such like.

For declaration whereof, firste you shall marke, whether your fraction be a simple fraction, other els a fraction of sundry partes, I meane of more tearmes than two. And if your

fraction

fraction bee a fraction of fractions, or other-wayes compounde, you must reduce it to one simple fraction. And then marke well the denomination of that other fraction into whiche you woulde turne this, for by that denominatour you must multiplie the Numeratour of youre firste fraction, and the totall producte thereof shall you diuide by the Denominator of your firste fraction, and that quotient shall be the Numeratour to the Denominator proponed: as for example, I haue this fraction $\frac{3}{5}$, whiche I woulde turne into tenth partes, therfore I multiplie this 10 by 3, that is the numeratour of my fraction, and there ryseth 30, which I diuide by 5, and the quotient is 6, which must be the numeratour to 10, and so $\frac{3}{5}$ will be $\frac{6}{10}$.

Scholer. This is easie ynoughe to do.

Mayster. Then shall you see another example of the same Fraction that is not so easie: as if I woulde turne $\frac{3}{5}$ into viij. partes, proue you that worke.

Scholer. I must multiply 8 by 3, and there amounteth 24: which I diuide by 5, and the quotient is 4, then is the new Fraction $\frac{4}{8}$.

Mayster. And see you nothing doubtfull in this worke?

Scholer.

Scholer. I see, that when 24 was diuided by 5, there remayned 4, whiche I did not passe of, bycause ye speake nothing of any remainder, but onely of the quotient.

Mayster. By likelyhoode you remember what I sayde to you in Diuision of whole numbers, that you shoulde not passe of the remaynder there, but onely note it as a summe that coulde not bee diuided without knowledge of Fractions. Wherefore nowe marke this, that in all diuisions of whole numbers, when there is any remayner, you shall set it ouer a line as a Numeratour, and set the Diuisor for the Denominatour, and that Fraction doth make the Diuision complete, and is parte of the quotient: as if I woulde diuide 48 by 5, the quotient will bee $9\frac{3}{5}$: so in youre former worke when 24 was diuided by $\frac{2}{5}$, the quotient shoulde be $4\frac{4}{5}$, and so the newe Fraction should bee thus: that is $\frac{4}{8}$ of the entier number, and $\frac{4}{5}\frac{1}{8}$: whiche you may proue by example of some coyne.

$$\frac{4\ \frac{4}{5}}{8}$$

Scholer. Then I take a Crowne, whose $\frac{3}{5}$ is 3 ß. Now if I would proue whether 3 ß bee $\frac{1}{5}$, I shall haue a comberous worke to do.

$$\frac{4\ \frac{4}{5}}{8}$$

Mayster.

Mayster. In deede for whole pennies your example is troublesome: yet turning the Crowne into halfe pennies, it is easie ynough.

Scholer. Now will I do it.

Mayster. First let me tel you an easie way howe to finde any number that will easily bee diuided into such partes as you desire, which way is this. Set downe the partes that you desire, and then by one of them multiplie all the other, the totall whereof shall conteyne all the partes proponed. As if I woulde haue a number that maye bee diuided into 4, 5, 6, and 7 partes, by 4 multiplie 5, and there ryseth 20: then multiplie 20 by 6, and it will make 120: which multiplyed by 7, will yelde 840: and so of any other numbers.

Scholer. Then in oure former example where is mention but of 5 parts, and 8 partes, I shall onely multiplie 5 by 8, which maketh 40, and that number will serue.

Mayster. So will it.

Scholer. Then what is $\frac{3}{5}$ of 40?

Mayster. Proue by the same rule whiche you confesse easie ynoughe: 3 times 40, is 120, which beeing diuided by 5, maketh 24: and that is iust.

$$\frac{4 \quad \frac{4}{5}}{8}$$

Nowe

Now to knowe whether it be equall to 24, firste I see by the same rule, that $\frac{4}{8}$ is 20, and $\frac{1}{8}$ is 5, of which 5 I must take $\frac{4}{5}$: and that by the same rule is 4. So that I see nowe, that is equall to $\frac{3}{5}$.

$$\frac{4\,\frac{4}{5}}{8}$$

Mayster. And by the way note this forme of Fraction howe it is written, that is to say, bothe the Numerator and his Fraction aboue the line: although I knowe it may bee written otherwayes, as thus $\frac{4}{8}$ and $\frac{4}{5}\,\frac{1}{8}$, but I accompte the other waye more apte a greate deale.

And so may you expresse by an other way, than is before mentioned, all Fractions of Fractions, as thus. That is $\frac{1}{4}$ of $\frac{5}{8}$, and so of other, but I remitte these fournies to the arbitrement of euery wise artes man, to vse as he thinketh most apt and readie.

$$\begin{matrix}\frac{1}{4}\\[4pt]\frac{5}{8}\end{matrix}$$

But nowe one example more for this rule, and then shall wee ende it. If I haue $\frac{7}{15}$ of a Soueraigne (accompting the Soueraigne 20 shillings) how many shillings is that $\frac{7}{15}$?

Scholer. I must multiply 7 by 20, and that maketh 140, which I shall diuide by 15, and the quotient will be $9\frac{5}{15}$: or else in lesser

termes

termes, $\frac{1}{3}$.

Mayster. That is 9 ß, and one third parte of a shilling, that is 4 d, as by this same rule you may proue. And this for this tyme shall suffise for Reduction, saue that I muste nowe repeate a little touching Multiplication and Diuision, and so go forwarde.

MVLTIPLICATION.

IN Multiplication it happeneth sometyme, that there bee whole numbers to bee multiplyed wyth Fractions: And maye bee in two sortes, for either the whole number is seuerall from the fraction, and is the multiplyer, or els, the whole number is ioyned with one or bothe of the Fractions, and so maketh a mixt number thereof. If it be in the first sorte, then needeth there no Reduction, but onelye multiply the Numerator of the Fraction by that whole number, and the totall thereof set for the new Numerator. Reduction of vvhole numbers into fractions.

Scholer. I vnderstande you thus. If I haue $\frac{6}{21}$, to bee multiplyed by 16, then muste I multiplye that 16 with 6, which is the Numerator,

merator, whereof commeth 96, and that muste I set for the newe Numerator, keeping styll 23 for the Denominator, and so the Fraction will bee $\frac{96}{23}$, that is $4\frac{4}{23}$.

Maister. And in this sorte of woorke you maye abbridge the labour, thus. If it happen the denominator to be suche a number, as may euenly bee diuided by the sayde whole number proposed, then diuide it thereby, and set the quotient of that Diuision for the former Denominator: but reserue styll the Numerator, and so is the Multiplication done.

Scholer. Then I fayne this example, $\frac{7}{20}$ to be multiplyed by 5. And bicause 5 will iustlye diuide 20, therefore I take the quotient of that diuision which is 4, and set in steade of 20, and so the Fraction will bee $\frac{7}{4}$, that is $1\frac{3}{4}$.

Mayster. Which is all one with $\frac{35}{20}$ that would haue folowed of the other sort of work.

Sch. I perceyue it very well.

Howve to multiply mixt numbers.

Mayster. Nowe then for the other sorte where the number is myxte, take this waye: firste to reduce the sayde whole number and fraction into one fraction Improper (as I shewed you in Reduction) and then multiplye them togyther, as yf they were proper fractions,

fractions.

Scholer. $13\frac{3}{5}$ beyng set to bee multiplyed by $\frac{5}{8}$, firste I muste reduce the myxte number by multiplying 13 by 5, and that maketh 65, whereto I muste adde the Numerator 3, and so the fraction will bee $\frac{68}{5}$, which now I shall multiplye after the accustomed forme, and it will bee $\frac{340}{40}$.

Mayster. You haue done well: and so may you see, that although most part of the formes of Multiplication may bee wrought without Reduction, yet some can not, as namely Myxed numbers.

And yet one note more will I tell you of Multiplication, before wee leaue it: That is, when so euer you woulde multiply any fraction by 2, which commonly is called Duplation, you may doe it not onely by doubling the Numerator, but also by parting the Denominator into halfe, if he be euen. Duplation

Scholer. Then if I woulde double $\frac{5}{12}$, I may chose whether I will make it, $\frac{10}{12}$, or els $\frac{5}{6}$. And in deede I see that all is one, but that the diuiding of the Denominator seemeth the better waye to make smaller tearmes of the Fraction, and so they shall neede the lesse Reduction.

Mayster. It is so: and now I shall not neede to tell you that Multiplication is proued by Diuision, and Diuision likewaies by multiplication, but the like workes that I shewed you in Multiplication, will I shewe you in Diuision also.

DIVISION.

Diuision to diuide a vvhole nūber by a Fraction.

When any whole number shall be diuided by a Fraction, you muste multiply the saide whole number with the Denominator of the fraction, and set the totall thereof for the new Numerator, and for the Denominator;, set the Numerator of the Fraction.

Scholer. Then 20 diuided by $\frac{3}{4}$ wyll make $\frac{80}{3}$.

To diuide a Fraction by a vvhol number.

Mayster. Euen so. But if you woulde diuide the fraction by the whole number, then multiply the Denominator by ye same whole number, and set the totall for the Denominator, without changing the Numerator.

Scholer. Then to diuide $\frac{20}{23}$ by 4, it will bee $\frac{20}{92}$.

Another

Mayster. You say well, And by the same

example

example you giue mee occasion to remember an other briefe way to doe the same: for if you had diuided the sayd Numerator by 4, and set the quotient for the Numerator, keeping still the olde Denominator, it woulde haue beene not onely as well done, but also in a Fraction of lesser termes. Breefvvay

Scholer. I gesse it to be euen so, by a like worke that you taught me in Multiplication. And for proofe thereof $\frac{20}{92}$ beeing the diuidende, and 4 the Deuisour, I diuide the Numerator 20 by 4, and the quotient is 5, which I set for 20 ouer 23, thus $\frac{5}{23}$. And I see that it is all one with $\frac{20}{92}$, as by diuiding bothe these tearmes by 4, and so reducing them to their leaste Denomination, I maye easilye proue.

Mayster. You conceyue it well. And if there be mixt numbers (other one or both) you must first reduce that mixte number into an improper Fraction. And then worke as you haue learned.

Scholer. That was sufficiently taught in Multiplication. Therefore I pray you go forwarde to some other thing.

Mayster. Then take this note yet for Diuision. If the Denominators be like, then di-

uide the Numerators as it if were in whole numbers, and the quotient whether it be Fraction, whole number, or mixt, is a good quotient for that Diuision. And generally if one of the Numerators maye iustly diuide the other, by that quotient multiplie the Denominatour of the lesser Numerator, and set it that doth amounte, in the roome of the same denominator, and then for a Numerator to it, set the denominator of the other fraction.

Scholer. Then if I woulde diuide $\frac{3}{4}$ by $\frac{12}{17}$ I see that 3 will diuide 12, and the quotient will be 4, by which I muste multiplie the other 4 that is the Denominator vnder 3, and then it is 16, whiche I set for the Denominator 4, and ouer it in steade of the 3, I must set 17, the other Denominator, and so is it thus, $\frac{17}{16}$.

Mayster. And so is $\frac{17}{16}$ in steade of $\frac{51}{48}$, whiche woulde haue rysen by the common worke.

Mediation. And nowe for Mediation (which is to diuide by 2) marke this: If the Numerator bee euen, sette the halfe of it in his place without the Diuisour, and so haue you done: and if the Numerator bee not euen, then double the Denominator.

Scholer.

Scholer. That is if I would mediate $\frac{6}{7}$, I may make the quotient $\frac{3}{7}$. And if I would mediate $\frac{7}{11}$ I must make it $\frac{7}{22}$.

Mayster. Now trust I that you haue sufficient knowledge in Reduction, Multiplication, and Diuision: and therefore will I goe in hande with Addition and Subtraction, which nowe will appeare easie ynough.

ADDITION.

When soeuer you haue any fractions to bee added, you muste consider whether they bee of one Denomination or not. And if they bee of one Denomination, then adde the Numerators togither, and set that amounteth, for the Numerator ouer the common Denominator, and so haue you done. The reason is, bicause that suche differ little in Addition or Subtraction from the worke of vulgare denominations, where the Denominators bee no numbers: as 3 pence & 5 pence, make 8 pence, where the Denomination is not altered. But

To adde Fractions of one denominatiō. 1

2. and if the Fractions bee not of one Denomination, or any of them be mixt of whole num-
3. bers and fractions, then muste you first reduce

To adde fractions of diuerse denominations.

them to one denomination, and after adde them. And if they be many, then adde firste
4. two of them, and to the summe that dothe amounte of the Addition, adde the thirde, and then the fourth, and so foorth, if you haue so many.

Scholer. This seemeth easie ynoughe, nowe that I haue alreadie learned to multiply and to Reduce, withoute whiche twoo, I coulde neuer haue wrought this. And therefore nowe I see good reason, why you did place Multiplication and Reduction before Addition.

Mayster. It is well considered, but yet refuse not to expresse your understanding of it, by an example.

Scholer. Then woulde I adde firste $\frac{7}{18}$ with $\frac{5}{18}$, and bicause the Denominators are like (and so needeth no Reduction) I adde 7 to 5, which maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbers $\frac{2}{3}$.

And if I haue many numbers to be added, as heere $\frac{7}{8}$ $\frac{4}{5}$ $\frac{9}{10}$, firste I muste reduce them (bycause they haue diuerse denominatours) into

to

to one Denomination, and then will they bee thus.

$\frac{150}{400}$ $\frac{180}{400}$ $\frac{160}{400}$

or in lesser Tearmes $\frac{15}{40}$ $\frac{18}{40}$ $\frac{16}{40}$, which by Addition we make $\frac{81}{40}$, that is $2\frac{1}{40}$.

Ma. Now may wee go to Subtraction.

SVBTRACTION.

Subtraction hath the same preceptes that Addition had, for if the Denominators bee lyke, then muste you subtracte the one numerator from the other, and the reste is to bee set ouer the common Denominator, and so youre Subtraction is ended: but and if you haue many Fractions to be subtracted out of many, then muste you reduce them to one Denomination, and into the seuerall Fractions, that is, all that must be subtracted into one Fraction, and the residue into an other Fraction, and then woorke as I sayde before.

Scholer. For the firste example I take $\frac{15}{12}$ to bee subtracted out of $\frac{17}{12}$, and the reste will be $\frac{2}{12}$ or $\frac{1}{6}$.

For an other example I take $\frac{3}{4}$ to bee sub-

trahed out of $\frac{7}{8}$, which I must first reduce, and it will be thus, $\frac{24}{32}$ and $\frac{28}{32}$.

Then doe I subtracte 24 out of 28, and there resteth 4, which I set ouer the common Denominator for a Remayner, thus, $\frac{4}{32}$, that is $\frac{1}{8}$.

Nowe for the thirde example, I take $\frac{3}{4}$ and $\frac{5}{8}$ to bee subtrahed from $\frac{7}{8}$ and $\frac{9}{10}$. And bycause their Denominators be diuers, I doe reduce them thus, $\frac{1440}{1920}$, $\frac{1600}{1920}$, $\frac{1680}{1920}$, $\frac{1728}{1920}$.

Then doe I adde the two firste, and they make $\frac{3040}{1920}$. Also I adde the two laste, and they yelde $\frac{3408}{1920}$. Then doe I subtract 3040 out of 3408, and there resteth 368, so is the remayner $\frac{368}{1920}$, that is in smaller Tearmes, $\frac{23}{120}$. And thus haue I done with Subtraction, except you haue any more to teach mee.

Mayster. Proue one example more of two Fractions of diuers Denominations.

Scholer. I take these two Fractions, $\frac{7}{8}$ and $\frac{9}{24}$, which being reduced, will stande thus, $\frac{168}{192}$, and $\frac{72}{192}$. Nowe woulde I subtracte 168 out of 72, but I can not.

Mayster. Then maye you perceyue that you mistoke the Fractions: for you can neuer subtract the greater out of the lesser, although you may adde, multiply or diuide the greater

with

with the lesser. And albeit that $\frac{7}{8}$ hath bothe his Tearmes lesser than $\frac{9}{24}$, yet is $\frac{9}{24}$ the lesser Fraction: for generally if you multiply the Numerators and Denominators of two fractions crosse wayes, that Fraction is the greatest, of whose Numerator commeth the greatest summe, as in this example: 7 multiplyed by 24, maketh 168: and 9 being multiplyed by 8, yeldeth but 72, therefore is the first Fraction $\frac{7}{8}$ the greatest of these two, so can you not subtract it out of a lesser Fraction. The greatest of two fractions.

But and you shoulde subtracte a Fraction out of a whole number, what would you doe?

Scholer. Marry I would reduce ẏ whole number into a Fraction of the same Denomination that my Fraction is, and then worke by Subtraction.

Mayster. So may you doe, but it is easier much, if your fraction be a proper fraction, that is to say, lesse than an Unite, to take an Unite from the whole number, and then turne it into an Improper Fraction, and so worke youre Subtraction. As yf I woulde subtracte $\frac{3}{5}$ from 4, I maye take one from 4, and tourne it into $\frac{5}{5}$, from which if I bate $\frac{3}{5}$, there will remayne $3\frac{2}{5}$. And if the firste fraction bee an Improper fra-

 ction

ction, then maye I take so many Units from the whole number, that they may make an improper fraction greater than that first, and then worke by Subtraction: As if there bee proponed $\frac{10}{3}$ to bee subtrahed from 6, bycause $\frac{10}{3}$ is more than 3, & not so much as 4, I muste take 4 from 6, and turne them into thirdes, thus. $\frac{12}{3}$ then abate $\frac{10}{3}$, and there resteth $\frac{2}{3}$, so the whole remayner is 2 $\frac{2}{3}$. And thus will I make an ende of the workes of common Fractions for this tyme, not doubting, but you can apply them bothe unto the rules of Progression, and also unto the Golden rule, without anye other teaching than you haue learned before, which might seme tedious to repeate, saue that in some speciall diuersities, which be peculiar to Fractions, I can not ouerpasse, but instruct you somewhat by the way.

THE GOLDEN RVLE.

Therefore as touching the Golden rule for the placing of the 3 numbers proponed in the question, whereby to finde the thirde, & for the forme of their worke, with other like notes,

I referre you to that whiche you haue alreadie learned.

But this easie forme of working by fractions shall you note, ỷ if your three numbers bee fractions, for an apt worke and certaine, multiply the Numerator of the first number in the question, by the Denominator of the seconde. And all that againe multiply by the Denominator of the thirde number, and the totall thereof shall you keepe for to bee the Diuisor. Then multiplie the Denominator of the firste number by the Numerator of the seconde, and the whole thereof by the Numerator of the thirde, and the totall thereof shall be your diuidende. Now diuide this diuidend by ỷ diuisor which you founde out before, and that number shal be the iiij. number of the question whiche you seeke for: As in this example, If $\frac{3}{4}$ of a yeard of Veluet cost $\frac{2}{7}$ of a Soueraigne, (esteemed at 20 shillings) what shall $\frac{5}{8}$ cost?

Scholer. If it please you to let me make the answere, I would first place these three numbers, as I learned in whole numbers, thus. A question of Veluet.

$\frac{3}{4}$ $\frac{2}{7}$

$\frac{5}{8}$

And then according to your newe rule, I must multiplie 3 beeing Numeratour in the firste number, by three the Denominatour of the

the seconde, and thereof commeth 9, whiche I multiplie againe by 6, the Denominatour of the thirde number, and so haue I 54. whiche I keepe for the Diuisor, then multiplie I 4, the Denominator of the first, by 2, the Numeratour of the seconde, and there ryseth 8, whiche againe I multiple by 5, the Numeratour of the thirde, and it maketh 40: then must I diuide 40, by 54, and it will bee $\frac{40}{54}$ that is $\frac{20}{27}$, in lesser tearmes, and then the figure will stande thus.

But what that is in money, I can not tell excepte I shall worke it by Reduction, as you taughte mee.

$$\begin{array}{ccc} \frac{1}{4} & & \frac{2}{3} \\ \frac{5}{6} & & \frac{20}{27} \end{array}$$

Mayster. It forceth not nowe, you maye reduce it when you liste, but it were disorderly done heere to mingle diuerse workes togither, where wee do not seeke the value of the thing in common money, but in an apt nūber, whiche you haue well done. And therefore will I yet shewe you an other like waye of easinesse in worke, howe you may chaunge your iij. Fractions into 3 whole numbers, by whiche you shall worke as if the question were proponed in whole numbers. The first number you shall

finde

finde as I taught you: nowe to finde the Diuiſor for the ſeconde number, take the Numerator of the ſeconde fraction: and for the third number take that, that riſeth of the Multiplication of the Denominator of the firſte, by the Numerator of the thirde, and then worke your queſtion.

Scholer For example heereof, I put this queſtion, If $\frac{11}{12}$ of 1 ℔ waight of ſiluer, bee worth $\frac{12}{4}$ of a Soueraigne, what is $\frac{1}{2}$ of 1 ℔. weight worth? For the anſwere, firſt I place the Fractions in order, thus.

$$\frac{11}{12} \quad \frac{12}{4} \quad \frac{1}{2}$$

Then to turne theſe Fractions into whole numbers, I multiple 11 whiche is the Numerator of the firſte, by 4, (the denominator of the ſeconde) and there commeth 44, whiche I multiply by 2 the Denominator of the thirde, and ſo amounteth 88, whiche I ſet for the Diuiſour in the firſt place. Then in the ſeconde place I ſet 12, whiche is Numeratour in the ſeconde Fraction, and in the thirde place I ſet the ſumme that amounteth of 12, beeyng the Denominatour in the firſte number, multiplyed by 1, beeing numerator in the thirde number, and ſo the figure will ſtande as heere you ſee

$$88 \quad 12 \quad 12$$

Then

Then to worke it forth, I multiply 12 by 12, and there amounteth 144, which I diuide by 88, and the quotient will bee $1\frac{56}{88}$, or in lesser termes, $1\frac{7}{11}$, and then the figure will stand thus.

$$\begin{array}{ccc} \frac{11}{12} & & 1\frac{1}{4} \\ \frac{1}{2} & & 1\frac{7}{11} \end{array}$$

Ma. These ij. formes now you vnderstand well ynoughe:

And as for any other, at this time I will not repeat, only this shall you marke for the profe of this rule, whether youre worke bee well wrought or no. Multiplie the firste number by the fourthe, and note what amounteth: then multiplie the seconde by the thirde, and marke what amounteth also. Nowe if those two numbers so amounting be equall, then is your worke well done, else you haue erred. And this shall suffice for the former rule, but in the Backer rule, this shall you note for ease of worke, that you multiplie the Numerator of the first by the Numerator of the second, and the whole thereof by the Denominator of the thirde, and that amounteth thereof, shall be the Diuidend. Then multiplie the Denominator of the firste by the Denominator of the seconde, and that whole by the Numerator of the third, and that riseth thereof shall be the Diuisor. Example of this: I did lende my friende $\frac{3}{4}$ of a Porte-

[The profe of the Golden rule.]

[The Backer rule.]

guise

guise vij. monethes, vppon promise that hee should doe as much for mee againe: and when I should borowe of him he could lende me but $\frac{5}{12}$ of a Porteguise, nowe I demaunde how long tyme muste I keepe his mony in iuste recompence of my lone, accompting 13 monethes in the yeare? A question of lone.

Scholer. The first number must be the first monye borowed, that is $\frac{3}{4}$ of the Porteguise: the second number the 7 monethes, that is $\frac{7}{13}$ of a yeare: & the thirde number the mony that was lent in recompence, that is $\frac{5}{12}$ of a Porteguise: then I set the numbers, thus.

$$\frac{3}{4} \; Z \; \frac{7}{13}$$
$$\frac{5}{12}$$

Then (as you taught me) I multiplye 3 (beeing Numeratour in the firste number) by 7 the Numeratour of the seconde number, and it maketh 21, which I multiply by 12 the Denominator of the thirde, and so haue I 252 for the diuidend: then I multiply 4 the denominator of the firste, by 13 the denominator of the seconde, and it yeldeth 52, which I multiply agayne by 5, the Numeratour of the thirde, and it will make 260, that is the diuisor. Then muste I diuide 252, by 260, so it will bee in the smallest fraction, $\frac{63}{65}$ of a yeare.

Maister.

Maiſter. And this doe you ſee ſome eaſe in woorking, better than to multiply and diuide tediouſlye ſo many Fractions. An other queſtion yet will I propoſe, to the intent you may ſee thereby the reaſon of the ſtatute of aſſiſe of breade and ale, which in all Statute bookes in Frenche, Latine, and Engliſhe, is muche corrupted for wante of knowledge in this arte: for the right vnderſtanding whereof I propone this queſtion.

Statutē of Aſſiſe of breade and ale.

Queſtion. When ỷ price of a quarter of wheate is 2 ß. the farthing white lofe ſhall weye 68 ß: then I demaunde, what ſhall ſuche a loafe weye, when a quarter of wheate is ſolde for 3 ß?

Scholer. This queſtion muſte be wrought as it is proponed in whole numbers and not in Fractions.

Mayſter. You ſeeme to ſaye reaſonable, how bee it, in that Statute of Aſſiſe, the rate is made by the proportion of partes in a pounde weyght Troye, els could it not bee a Statute of any long continuaunce, ſeeing the ſhillings doe chaunge often, as all other monies doe: but this Statute beyng well vnderſtanded, is a continuall rule for euer, as I will anone declare by a new table of Aſſiſe, concerning the ſhillings into vnces and partes of vnces.

Therefore

Therefore here by a shilling you muste vnder- Note vvhat a shilling is
stande $\frac{1}{20}$ of a pounde weight, and so by pens $\frac{1}{20}$ of an vnce, wherefore although you might worke this question proponed by whole number well ynough, for that time when ẏ statute was made, yet to apply it to our tyme, and to make it to serue for all tymes generallye, it is beste to worke it by fractions, setting for 2 shilling $\frac{2}{20}$: and for 68 shillings, $\frac{68}{20}$: and so for three shillings $\frac{3}{20}$, and then will the figure of the question stand thus.

$$\frac{2}{20} \quad \frac{68}{20}$$
$$\frac{3}{20}$$

In which question, bicause all the denominators bee like, you shall worke onely with the numerators.

Scholer. Then I shall multiply 68 by 2, whereof commeth 136, which if I diuide by 3, the Quotient will bee $45\frac{1}{3}$: but howe shall I make a Fraction of that to stande with the other?

Mayster. Haue you so soone forgotten what was taught you so lately? This is his forme.

$$\frac{45\frac{1}{3}}{20}$$

Sch. I remember it nowe and then it signifieth 45 twenty partes, and the thirde deale of one twentye parte.

Mayster. So is it, and that maketh in

shillings, 45 shillings, foure pence: whereby you may note one greate error in the Statute bookes, which haue constantly 48 shillings in that Assise. And by this rule, if you examine the statute, you shall finde many summes false, wherefore for the true vnderstanding of that statute & such like as I haue made mention of it, and somewhat recognised it, so doe I wish that all gentlemen and other studentes of the lawes, would not neglect this arte of Arithmetike as vnneedefull to their Studies. Wherefore to encourage them thereto, and to gratifie bothe them and all other in generall, I will heere exhibite a Table of that parte of the statute in two columnes, and in a third columne I will adde yͤ correction of those errors which haue crept into it.

Here followeth the Table.

Here ffollowethe the Table

Henry Sparstowe

Henry Sparstowe

The price of a quarter of vvheate.		The vveight of a farthing vvhite lofe by the statute bookes.			The correction by iust Assise		
s.	d.	li.	s.	d.	li	s.	d.
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 $\frac{1}{5}$	2	14	4 $\frac{4}{5}$
3	0	2	8	0	2	5	4
3	6	2	2	0	1	10	10 $\frac{2}{7}$
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 $\frac{2}{3}$
5	0	1	8	2 $\frac{1}{2}$	1	7	2 $\frac{2}{5}$
5	6	1	4	8 $\frac{1}{4}$	1	4	8 $\frac{8}{11}$
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 $\frac{1}{13}$
7	0	0	19	1	0	19	5 $\frac{1}{7}$
7	6	0	18	1 $\frac{1}{2}$	0	18	1 $\frac{1}{5}$
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 $\frac{1}{4}$	0	15	1 $\frac{1}{3}$
9	6	0	14	4 $\frac{1}{4}$	0	14	3 $\frac{15}{10}$
10	0	0	13	7 $\frac{1}{2}$	0	13	7 $\frac{1}{5}$
10	6	0	12	11 $\frac{1}{2}$	0	12	11 $\frac{1}{7}$
21	0	0	12	4 $\frac{1}{4}$	0	12	4 $\frac{4}{11}$
11	6	0	11	10	0	11	9 $\frac{21}{23}$
12	0	0	11	4	0	11	4

In the common bokes there is no farther rate of assise made, than unto 12 ſ the quarter of wheate: but in an auncient copie of 200 yeares oulde (which I haue) ther is added the rate of assise unto 20 ſ the quarter, but yet was ÿ assise also eyther wrong cast at ÿ first pening, or els corrupt sith that time, for lacke of wise knowledge in the rule of proportion, whiche I will adde here also, to gratifye suche as be studious in thys sorte of knowledge, and desire to understande truth exactly.

The price of a quarter of wheate.		The weight of the farthing white lofe by the statute bokes.		The correction of the errors.		
ſ.	d.	ſ	d.	ſ.	d.	
12	6	11	0	10	10	14/25
13	0	15	0 1/2	10	5	7/13
13	6	10	1 1/2	10	0	8/9
14	0	9	7	9	8	4/7
14	6	9	2 1/2	9	4	16/29
15	0	9	1 1/2	9	0	4/5
15	6	9	1 1/4	8	9	9/11
16	0	9	0	8	6	0
16	6	8	6	8	2	10/11
17	0	8	3	8	0	0
17	6	7	10	7	9	9/13
18	0	7	6	7	6	1/7
18	6	7	3	7	4	9/17
19	0	7	2	7	1	17/19
19	6	5	10	6	11	9/13
20	0	5	6	6	9	1/7

These ij. tables I haue set seuerall, bicause no man should thinke that I would either adde or take away from any law those partes whiche might of right seeme either superfluous other diminute, but yet I may not bee so curious as to neglect manifest errours, which is not onely my part, but euery good Subiectes dutie with sobriety to correct And for auoyding of offence I haue rather done it in this priuate booke, rather than in any booke of the statutes selfe, trusting that all men will take it in good part.

Sc. I woulde wishe so, but I dare not hope so, sith neuer good man that would reforme errour, could escape ẏ benimous tonges of enuious detractors, which bicause they either cānot or liste not do any good them selues, do delite to bark at the doings of other, but I beseke you to stay nothing for their peruerse behauiour.

Ma. I consider many things ẏ some may obiect, whereunto I am not vnprouided of iust answers, but I wil not seeme so hasty to make the answers before I heare their obiectiōs. but as I trust that men are of a better nature, and more grateful now thā some hath bin in times passed, as I haue done in ẏ statute of Assise for bread in rate of shillings, so will I set forth the like table in poundes & ounces, a[illegible]artes

thereof, ẏ it may be easily applied to all times: but I meane not by this to alter any worde of ẏ Statute (being so good an ordinance & of so great continuance) but onely to make it as a kind of expositiō & declaratiō of ẏ said statute, trusting ẏ therby the statute may be better vnderstand, & consequently better put in execution. And here you shall note, ẏ I haue accompted the shillings after the rate of lx ß, to ẏ poūd weight, bicause I esteme it ẏ most apt rate for our time. Wherfore if in the first columne you find the price of wheat, directly agaynst it in the second columne, you may finde ẏ weight of the farthing white lofe, in this our time, and if you double ẏ number (as I haue done in the thirde columne) then haue you the weight of the halfe pēny white lofe, & so in the fourth colūne is set the weight of ẏ pēny white lofe. It needeth not to tell you that, ẏ the sight doth testifie, howe ẏ euery colūne is parted into thre smaller pillers, wherof ẏ first colum hath these thre titles. poūdes, shillings, & pennies: ẏ other three columnes haue ech of thē these three titles, pounds, vnces, & penny weights. And as in the first colūne xij. pens make a shilling, & xx. ß. maketh a pound, so in ẏ other iij colūnes xx. pēny weight maketh an vnce, and xij. vnces do make a pound.

A pounde vveight.

The

l.	s.	d.	li.	unc.	d.w.
0	3	0	6	9 1/2	2
0	4	6	4	6 1/4	3
0	6	0	3	4 3/4	1
0	7	6	2	8 1/2	2 4/5
0	9	0	2	3	4
0	10	6	1	11 1/4	1 2/7
0	12	0	1	8 1/4	3
0	13	6	1	6	2 2/3
0	15	0	1	4 1/4	1 2/5
0	16	6	1	2 1/4	1 9/11
0	18	0	1	1 1/2	2
0	19	6	1	0 1/2	1 1/13
1	1	0	0	11 1/2	3 1/7
1	2	6	0	10 3/4	2 1/5
1	4	0	0	10	4
1	5	6	0	9 1/2	2
1	7	0	0	9	1 1/3
1	8	6	0	8 1/2	1 15/19
1	10	0	0	8	3 1/5
1	11	6	0	7 1/2	0 3/5
1	13	0	0	7 1/4	3 4/11
1	14	6	0	7	1 21/23
1	16	0	0	6 2/3	1
1	17	6	0	6 1/2	0 14/25
1	19	0	0	6 1/4	0 7/13
2	0	6	0	6	0 8/9

The price of a quarter of wheate.

The weight of the farthing white lofe.

The weight of the halfe penny white lofe.

li.	vnc.	d.w.
13	7	4
9	0 1/4	1
6	9 1/2	2
5	5 8/16	0 3/5
4	6 1/4	3
3	10 1/2	2 4/7
3	4 1/4	1
3	0 1/4	0 1/3
2	8 1/2	2 4/5
2	5 1/2	3 5/11
2	3	4
2	1	2 2/15
1	11 1/4	1 2/7
1	9 1/4	0 1/5
1	8 1/4	3
1	7	4
1	6	2 2/3
1	5	3 1/15
1	4 1/4	1 2/5
1	3 1/2	0 6/7
1	2 1/4	1 8/11
1	2	3 19/23
1	1 1/2	2
1	1	1 7/25
1	0 1/2	1 7/13
1	0	1 7/9

The weight of the penny white lofe.

li.	vnc.	d.w.
27	1/4	3
18	1 1/2	2
13	7	4
10	10 1/2	1 1/5
9	0 1/4	1
7	9 1/2	0 1/7
6	9 1/2	2
6	0 1/2	0 2/3
5	5 1/4	0 3/5
4	11 1/2	1 10/11
4	6 1/4	3
4	2	4 4/13
3	10 1/2	2 4/7
3	7 1/2	0 2/5
3	4 1/4	1
3	2 1/4	3
3	0 1/4	0 1/7
2	10 1/4	2 3/19
2	8 1/2	2 4/5
2	7	1 5/7
2	5 1/2	3 5/11
2	4 1/4	2 15/23
2	3	4
2	2	2 6/25
2	1	2 2/13
2	0	3 5/7

Scholer. Sir, I we thanke you most hartely for this, not onelye in myne owne name and in the name of all studentes, but also in the name of the whole Commons, to whome the restitution of this Assyse (I truste) shall bring restitution of the weyghte in breade, which long time hath been abused. And if you knowe anye like things more, wherein you woulde vouchsafe to declare the errours and set forthe the truthe, you can not but obtayne greate thākes of all good harted men that loue the common wealth.

Mayster. I haue sundrye things to declare, but I haue reserued them for a priuate booke by it selfe, yet not withstanding bycause the statute of the rate of measuring of ground is so common that it toucheth all men, and yet no more cōmon than needefull, but so much corrupt, that it is to farre out of all good rate, not onelye in the Englishe bookes of statutes commonlye printed, but also in the Latine bookes, and in the Frenche also, for I haue readde of eche sorte, and conferred them diligently, I will giue you a Table for the restitution of those errours, as may suffise for this present tyme. And firste will I propose one question to you touching the vse of that Sta-

 tute,

tute, whereby you may perceyue the order how to examine the whole Statute, and euery parcell thereof, and the question is this.

A question of measure of groũde.

When the Acre of grounde dothe contayne foure perches in breadthe, then muste it contayne 40 perches in lengthe: then doe I demaunde of you, howe much shall the lengthe of an Acre bee, when there is in the breadthe of it 13 perches? But before you shall aunswere to this question, I will declare vnto you an other Statute, which is the grounde of the former Statute. And that Statute is this. It is ordeyned, that 3 Barlye cornes, drye and rounde, shall make vp the measure of an ynche: xij. ynches shall make a foote, and iij. foote shall make a yarde, (the common Englishe bookes haue an elne) fiue yardes and a halfe shall make a pearche, and fortye perches in lengthe, and iiij. in breadthe, shall make an Acre. This is that Statute: whereby you may perceyue, that the intent of the Statute is, that one Acre should contayne 160 square perches. Nowe let mee heare you answere to the question.

A statute of measures

An Acre.

Scholer. As I perceyue by the wordes of that Statute, a perche to bee $\frac{1}{160}$ of an Acre; so coulde I make those numbers all in Fra-

ctions

tions, and so worke the question : but seeyng I may doe it also in whole numbers, I take that forme for the most easie, therefore thus I set the question in forme. Then do I multiply 40 by 4, and it maketh 160, which I diuide by 13, and the quotient is $12\frac{4}{13}$.

4 40
13

Mayster. Nowe turne that $\frac{4}{13}$ into the common partes of a perche, as they bee named in the former Statute : howe be it, it shall bee best to take one of the least partes in Denomination for auoyding of muche laboure, as feete, whereof the perche contayneth $16\frac{1}{2}$.

Scholer. Then to turne $\frac{4}{13}$ into feete, I Multiplye $16\frac{1}{2}$ by 4, and it maketh 66, whiche I muste diuide by 13, and the quotient is $5\frac{1}{13}$.

Mayster. So I finde that if the acre holde in breadthe xiij. perches, it shall contayne in length 12 perches, 5 foote, and $\frac{1}{13}$ of a foote, whiche is not fully an ynche, for the ynche is $\frac{1}{12}$ of a foote. But heere all the statute bookes in Latine and Englishe (that I haue seene) doe note it to be 13 perches, 5 foote and 1 inche: whiche maketh aboue 13 perches to manye in the acre, so that I woulde haue thought the errour

Note this errour.

erroure to haue crepte into the printed bookes by the greate negligence that Printers in our time doe vse, saue that in written Copies of great antiquitie, I doe finde the same. Yet haue I one Frenche Copie, which hathe xij. perches $\frac{1}{4}$, and one foote, and that misseth verie little of the truth.

Scholer Then I see it is true that I haue often hearde saye, that the truest Copies of the statutes be the French Copies.

Mayster. That is often true, but not generally, as I haue by conference tryed diuersly: but in this statute the French booke is most corrupt in all other places lightly.

But nowe to performe my promise, I will set forthe the Table for measuring of an Acre of grounde onely by suche partes as the Statute dothe mention, bicause at this time I doe of purpose wryte it for the better vnderstanding of the Statute, and hereafter with other things I intende to set forth this same more at large.

In this table following, I haue not done as in the other statute before compared by restitution with the faultes crepte into the Statute, but onelye haue written that true measure, which the equitie of the Statute dothe

pretende

pretende. For it were to vyle to iudge of so noble Princes and worthie Councellours as haue authorised and set forthe this statute, that they would make one acre in any forme greater than an other, but euery one to bee iust and equall with ech other, whiche is the grounde also of my worke, and heereby maye all men perceyue howe needefull Arithmeticke is vnto the Studentes of the lawe. But now I thinke beste to make an ende of these matters for this present time, sithe the Table hathe in it none obscuritie, that I shoulde neede to declare.

The breadth.	The length of the acre.		The breadth	The length of the acre.	
perche.	perche.	feete.	perche.	perche.	feete.
110	416	0	28	5	11 $\frac{11}{14}$
11	14	9	29	5	8 $\frac{31}{58}$
12	13	5 $\frac{1}{2}$	30	5	5 $\frac{1}{2}$
13	12	5 $\frac{2}{13}$	31	5	2 $\frac{41}{62}$
14	11	7 $\frac{1}{14}$	32	5	0
15	10	11	33	4	14
16	10	0	34	4	11 $\frac{11}{17}$
17	9	6 $\frac{27}{34}$	35	4	9 $\frac{3}{7}$
18	8	14 $\frac{2}{3}$	36	4	7 $\frac{1}{3}$
19	8	6 $\frac{18}{19}$	37	4	5 $\frac{11}{17}$
20	8	0	38	4	3 $\frac{9}{19}$
21	7	10 $\frac{3}{14}$	39	4	$\frac{9}{13}$
22	7	4 $\frac{1}{2}$	40	4	0
23	6	15 $\frac{18}{23}$	41	3	1 $\frac{73}{82}$
24	6	11	42	3	13 $\frac{15}{42}$
25	6	6 $\frac{2}{5}$	43	3	11 $\frac{77}{86}$
26	6	2 $\frac{7}{13}$	44	3	10 $\frac{1}{2}$
27	5	15 $\frac{5}{18}$	45	3	9 $\frac{1}{6}$

Scholer.

Scholer. In deede Sir, I vnderstand the Table (as I thinke) by those other which you set forthe before. For in the firste columne is set the perches of the breadthe of anye Acre, and then in the 2 columnes following appeareth howe many perches and howe manye foote the same Acre muste haue for his length.

Maister. You take it well: how bee it to speake exactly of breadthe and lengthe, the first columne dothe sometime betoken the breadthe, and sometime the length, for properly the longest side of any square dothe limit his length, and the shorter side doth betoken the breadthe: yet it is no greate abuse in suche tables, where a man can not well chaunge the title, to let the name remayne, although the proportions of the numbers we chaunge: for still by the first colũne, is expressed the measure of the one side, and by the two other pillers in one columne, is set forthe the measure of the other side. And this shall bee sufficient nowe for the vse of the Golden rule.

Nowe somewhat wyll I touche certayne other rules, which for their seuerall names maye seeme diuerse rules and distincte from

from this, but in deede they are but braunches of it: yet bycause they haue not onely seuerall workings in appearaunce, but also pleasant in vse, I will giue you a taste of eche of them. As for the rule of felowshippe, bothe single and double, with tyme and without tyme, I shall neede to saye little more than I haue already sayde in teaching the workes of whole numbers, yet an example or two will wee haue to refreshe the remembrance of the same, and to declare certayne proper vses and applications of it, as this for one.

A question of vnequall societie.

Foure men gette a booty or prise in tyme of warre, the prise is in value of mony 8190 li, and bicause ye men be not of like degree, therefore their shares maye not bee equall, but the chiefest person will haue of the bootye the thirde parte, and the tenthe parte ouer: the seconde will haue a quarter and the tenthe parte ouer: the thirde will haue the sixt parte: and so there is lefte for the fourthe man a verye small portion, but suche is his lot, (whether hee bee pleased or wroth) hee must be content with one xx. parte of the praye. Nowe I demaunde of you, what shall euerye man haue to his share?

Scholer. You muste bee fayne to answere

to

to your owne question, els is it not like to bee answered at this tyme.

Maister. The forme to understande the solution of this question, and all such like, is this: Reduce all the Denominators into one number by Multiplication, except that any of them be partes of some other of them, for all such partes you may ouerpasse, and take for them all those nūbers, whose partes they be: as in this example ẏ shares bee these $\frac{1}{3}$ $\frac{1}{10}$, $\frac{1}{4}$ $\frac{1}{10}$, $\frac{1}{6}$ $\frac{1}{20}$, if I multiply all the Denominatours together, beginning with 3, and so go on unto 20, it will make 144000: but considering that 3 is a parte of 6, I shall omitte that 3, and likewayes 10, which is a parte of 20, I may ouerpasse also, and then is there but 3 denominatoures to multiply, that is 4, 6, and 10, which make 240, which summe I take for my worke, bycause all the denominatours will bee founde in it. Then I take such partes of it as the question importeth, that is for the firste man, $\frac{1}{3}$ $\frac{1}{10}$, the $\frac{1}{3}$ is 80: the $\frac{1}{10}$ is 24: which I put in one summe for the firste mannes share, and it maketh 104. Then for the seconde mannes share, I take $\frac{1}{4}$, which is 60, and $\frac{1}{10}$ which is 24, and that maketh in the whole 84. Nowe for the thirde

 man

man which muste haue $\frac{1}{6}$ I take 40. And for the fourthe man there remayneth but 12, which is $\frac{1}{20}$ of the whole summe: so that if ẏ whole pray had been but 240 lb, then were the question answered: but bicause the summe was of greater value, by this meanes nowe shall I knowe the partition of it. I muste set my numbers by the order of the Golden rule, putting in the firste place the number that I founde by multiplying the Denominatours, and in the second place the summe of the boo-tye. And looke what proportion is betweene that firste number and that seconde, the same proportion shall bee betweene the partes of the firste number and the partes of the seconde, comparing eche to his like. Therefore I muste put in the thirde place, one of the partes or shares, and then worke by the former rule of proportion or Golden rule. And bycause I haue 4 seuerall partes of the firste number, by which I woulde finde out 4 like partes of the seconde number, therefore muste I make 4 seuerall figures.

The reason of this rule.

Scholer. Nowe I truste I can answere to your question, as by youre fauoure I will proue.

a		b	
240	8190	240	8190
104		84	

c		d	
240	8190	240	8190
40		12	

And to trie it, I set the 4 figures thus, marked with a, b, c, d, to shewe their order. And then in eche of them I multiplie the seconde number by the thirde, and diuide theyr totall by the first, and so amounteth the fourth summe whiche I seeke for, for if I doe multiply 8190 by 104, it maketh 851760, which beeing diuided by 240, maketh in the quotient 3549 for the firste mans portion. And so working with the other three figures, I finde for the seconde man 2866½, and for the thirde man 1365: and then for the fourthe man 409½. And so is euery mans share set forth in the figure here annexed.

a		b	
240	8190	240	8190
104	3549	84	2866½

c		d	
240	8190	240	8190
40	1365	12	409½

 And

And thus I thinke I haue done well.

Mayster. If you misdoubt your working and liste to proue it, adde all the shares togither: and if they make the totall, then seemeth it well done.

The proofe by Addition.

Sc. I may set them thus: and then by Addition the iuste summe dothe amounte, that is 8190, and therefore (as you saye) it seemeth to be well wrought.

3549
2866 $\frac{1}{2}$
1365
409 $\frac{1}{2}$
8190

The iust proofe.

But I beseeche you, is there any doubt in this triall, that you vse that worde, Seemeth?

Mayster. You may easily coniecture, that if you did assigne the firste mans share to the last, and so chaunge all the rest, that one had an others share, yet woulde the Addition appeare all one, and therefore is not the proofe exact.

But if you will make a iust proofe, for the firste mannes parte take $\frac{1}{3}\ \frac{1}{10}$ of the whole summe; and if it agree with the number in the figure, then it is well done. And so doe for the seconde, thirde, and fourth summes, and this proofe fayleth not. Nowe will I propounde certaine other questions whiche haue beene sette foorthe by certayne learned men, albeit not without some ouersighte, whiche

which questions I protest hartily I do not repeate to depraue those good men, whose labours and studies I much prayse and greatly delight in, but onely according to my profession, to seeke out truth in all things, and to remoue all occasions of errour, as muche as in me lyeth: and for that cause I will onely name the questions without hurting the Authours name. The first question is this.

Foure men did builde a house, which cost them 3000 Crownes, their shares were such, that one man shoulde paye $\frac{1}{2}$ of the summe, and 6 crownes ouer: the seconde should pay $\frac{1}{3}$, and 12 crownes ouer: the third man must lay out $\frac{2}{3}$, abating 8 crownes, and the fourth manne shoulde pay $\frac{1}{4}$, and 20 crownes more, can you answere to this question? A question of building.

Scholer. No in good faythe sir, and that you knowe best of any man, for I knowe no more than you haue taught mee.

Mayster. Then I dare saye you can not do it, neither yet ye best learned man that euer did propose it, for the question is impossible: for declaration whereof I will be bolde to vse firste the representation of the numbers in their aptest fourme, (althoughe I haue not yet taught you that manner of worke) by- An imposfible question.

cauſe it may appeare plainely that the queſtion is not poſſible, for here I haue ſet the partes, and added them, and they make the hole ſumme and $\frac{3}{4}$, and 30 more. Nowe howe is it poſſible to diuide truely

$\frac{1}{2}$	+	6
$\frac{1}{3}$	+	12
$\frac{2}{3}$	—	8
$\frac{1}{4}$	+	20
$1\frac{3}{4}$	+	30

eyther gaynes eyther charges ſo, that the particulars ſhall be more than the totall?

Scholer. It is agaynſt the forme of proofe by Addition of the partes.

Mayſter. You ſay truth. And bycauſe you ſhall perceyue it the better, I will trye it after the vulgare forme, as in this figure you ſee where the $\frac{1}{2}$ with 6 ouer is 1506: for the totall is as you hearde before 3000: the $\frac{1}{3}$ and the 12 more, is 1012:

1506
1012
6992
770
5280

the $\frac{2}{3}$ woulde bee 2000, but then abating 8, it is but 1992, and then laſte of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: whiche all beeing added in one ſumme, do make 5280, where ẏ totall ſum ſhould be but 3000, which ſum if you diuide by $\frac{4}{3}$, ſo ſhall you haue $\frac{3}{4}$ of it, that is 2250, & thereto adde 30 more, then

will

will those 3 summes make 5280: whereby you may see howe this forme as well as the other, dothe declare that the particulars in that question woulde make more than the whole summe by ¾, and 30 more: and therefore can that question not bee accepted as a possible thing, but yet doe certayne learned menne propounde suche questions, and answere to them. Therefore somewhat to saye to their excuse rather of theire good meaning then for their doyng, I will anone declare what may be saide for their defence: but in the meane season I will propounde the question as it maye bee wroughte by good possibilitye. As if foure men buyld a house togither, and it cost them 3000 crowues, and then for the partition they agree thus: that as often as the firste manne dothe paye 6 crowmes, so often the seconde shall paye 4, the thirde man 8, and the fourthe man 3. Or els thus: that the firste man shall pay double so muche as the iiij. and the seconde man shall paye $\frac{2}{3}$ of the firste mans charge: the thirde man shall paye double so muche as the seconde: (And these two wayes are to one ende) but farther for their agreement it is appoynted also, that the firste man shall gyue 6

3000
2250
30
5280

 crowmes

crownes ouerplus, and the seconde 12, and the iiij. shall gyue 20, but the third man shall gyue no ouerplus, but shall haue 8 crownes abated of his charge. Now is the question possible to be soyled, and this is the way to doe it. Marke the proportion of the seuerall charges, and set out small numbers in that rate, by which you may reduce the worke to the Golden rule, as here in the first forme, the numbers are already named, 6, 4, 8, 3: and in the seconde forme, although they be not playnely named, yet they may be the same numbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and agayne 8 is double to 4. Now adde these togither, and they make 21, which 21 muste bee set in the firste number in the Golden rule: for if it with the ouerplus of eche mannes charge woulde make the totall summe of the charges, then were those seuerall summes, the charges of eche man, beside hys ouerplus: but nowe it is not so.

The rule. But yet this is trewe, that looke what proportion eche of these seuerall summes dothe beare to 21, the same proportion doth the iuste charges of euerye man (beside his ouerplus) beare to the totall of the charges, the ouerplus being deducted: wherfore this may you note, that before you doe apply the totall of the charges

ges to the Golden rule, you muste deducte the ouerplus which is 6, 12, and 20, that is in the whole 38: but then 8 muste bee restored for the abatement of the thirde man, and then remayneth to bee deducted 30. Take 30 therefore out of 3000, and there will reste 2970, which I muste sette in the Golden rule for the seconde summe: and for the thirde summe I must put eche of the small numbers before mentioned, which although they bee not the seuerall charges, yet they represent them in proportion. And so making for euerye mannes charge a seuerall question, the figures will bee 4, which I marke with foure letters, a. b. c. d, thus.

a.

21 — 2970
6 — $848\frac{4}{7}$

b.

21 — 2970
4 — $565\frac{5}{7}$

c.

21 — 2970
8 — $1131\frac{3}{7}$

d.

21 — 2970
3 — $424\frac{2}{7}$

where I haue set for brieknes the summe of euery mãs charge in the fourth place, presupposing that you can tell howe to trye out that fourthe summe by so manye examples as yee haue had.

Scholer. As I truste that I doe vnderstande this forme, so I desire much to knowe what may be saide for them that mistooke this question.

Mayster. You seeme so desirous to knowe this errour, that you haue forgotten to examine whether this worke be without errour.

Scholer. Mee seemeth this woorke to bee well done, bycause the Addition of the 4 seuerall numbers dothe make the totall summe of 2970, which was to bee diuided into such foure partes.

Maister. But then haue you forgotten that the firste man muste paye 6 Crownes more besides this share, and the seconde man 12 crownes more: the thirde man 8 Crownes lesse: and the fourth man 20 Crownes more, for without these, youre firste totall of 3000 crownes will not be made.

Scholer. Then muste I adde to the firste mannes summe 6 more, and it will bee $85\frac{4}{7}$: and to the seconde sum I muste adde 12, and it wyll bee $577\frac{5}{7}$: from the thirde summe I must abate 8, and then will the summe bee $1123\frac{3}{7}$: then adding vnto the 4 summe 20, it wyll bee $444\frac{2}{7}$: & these 4 summes will make 3000, which is the whole charge, as in this example

ample it may appeare, where firſte I gather the $\frac{14}{7}$, that maketh 2, and ſo proceede I in the Addition to the ende.

$$
\begin{array}{r}
854\frac{4}{7} \\
577\frac{5}{7} \\
1123\frac{3}{7} \\
444\frac{2}{7} \\
\hline
3000 \\
\hline
\end{array}
$$

Mayſter. Now haue you well done, & this worke in the ſame ſummes is brought of other learned men for the true ſolution of the queſtion as it was firſt proponed, which as (I ſayde) was impoſſible: and nowe examine it by theſe ſeuerall ſummes, and ſee whether it do agree with the ſummes in the queſtion proponed.

The firſte man muſte pay $\frac{1}{2}$ and 6 ouer of the totall ſumme: howe thinke you, is $854\frac{4}{7}$ the halfe and 6 more of 3000?

Scholer. No that is it not, for it woulde be 1506: and for the ſeconde man 1012: and for the thirde man 1992: and for the fourthe man 770, whereof not one ſumme agreeth to this worke. But I maruaile that ſo wiſe men coulde be ſo much ouerſeene.

Mayſter. It is commonlye ſeene, that when men will receyue things from elder wryters, and will not examine the thing, they ſeeme rather willing to erre with their auncientes for companie, than to bee bolde to examine their workes or wrytings, whiche

ſcrupuloſitie

scrupulositie hath ingendered infinite errours in all kindes of knowledge, and in all ciuill administration, and in euery kinde of arte: but these learned menne did not meane anye other thing by this question, than to finde suche numbers as shoulde beare the same proportion togither, as those numbers in the question proponed did beare one to an other: which thing you shall perceyue more plainelye by an other question of theirs, that is this.

A question of a testament.

A man lying vppon his death bed, bequeatheth his goodes (which were woorth 3600 Crownes) in this sorte. Bicause hys wyfe was great with childe, and he yet vncertaine whether the childe were a male or a female, hee made his bequest conditionally, that if his wife bare a daughter, then shoulde the wyfe haue halfe his goodes, and the daughter $\frac{1}{3}$, but if she were deliuered of a sonne, thē that sonne shoulde haue $\frac{1}{2}$ of the goodes, and his wife but $\frac{1}{3}$. Now it chaunced hir to bring forthe bothe a sonne and a daughter, the question is: How shall they parte the goodes agreable to the testatour his will.

Scholer. If somme cunning Lawyers had this matter in scanning, they woulde determine

termine this Teſtament to be quite voyde, and ſo the man to die vnteſtate, bycauſe the teſtament was made vnſufficient, ſithe this condition was not expreſſed in it, and alſo it might haue chaunced that ſhee ſhoulde haue brought foorthe neyther ſonne nor daughter, as often hath bene ſeene, ſo is the will vnſufficiente in that pointe alſo.

Mayſter. Such ſcanners ſhoulde ſeeme to cunning, and yet not ſo cunning as cruell: for the mind of the Teſtatour is to be taken fauorably, for the ayde of the legatories, when there ryſeth ſuch doubts. But let vs trie this worke, not by force of lawe, but by proportion Geometricall, ſeeing the Teſtatour did minde to prouide for ech ſort of them.

Scholer. If the ſonne ſhall haue $\frac{1}{2}$ by force of the Teſtament, ſo muſt the mother haue $\frac{1}{3}$. Againe bicauſe ſhe hath a daughter alſo, therefore ought ſhee to haue $\frac{1}{2}$, and the daughter $\frac{1}{3}$: that is both wayes $\frac{1}{2}\ \frac{1}{3}$, and $\frac{1}{2}\ \frac{1}{3}$, which commeth to the whole goodes, and $\frac{2}{3}$ more. Wherefore it ſeemeth alſo impoſſible.

Mayſter. In this matter the minde of the Teſtatour is ſo to be vnderſtād, that ſuch proportion ſhoulde bee betweene the portion of the wyfe and the ſonne, as is betweene $\frac{1}{2}$ and

and $\frac{1}{2}$, that is, the sonne muste haue $\frac{3}{6}$ for $\frac{2}{6}$ to his mother, so shall hee haue 3 to 2, that is as much as his mother, and halfe as muche more: and the mother muste haue the lyke rate in comparison to hir daughter. Then muste I finde out 3 numbers in suche proportion, that the firste maye bee as muche as the seconde, and halfe as muche more (that is) in proportion *sesquialtera*, and the seconde to the thirde in the same proportion, suche numbers be 6,9,4.

Scholer. I pray you sir, how shall I finde out those numbers?

To finde three numbers in any proportion

Mayster. That will I gladly tell you. Whatsoeuer the proportion bee for any three numbers, multiply the Ternies of that proportion togither, and the number that amounteth, shall be the middle number of the 3: then multiply that middle number by the lesser Terme, and diuide that totall by the greater, and the least number of the 3 will amount. So if you multiplie that middle number by the greater extreme, and diuide that totall by the lesser extreme, then will the greatest number of that progression amount.

To find the proportion

Scholer. Then in this example, to finde the proportion of $\frac{1}{3}$ to $\frac{1}{2}$, I muste diuide (as you

you taught me in Diuision) $\frac{1}{2}$ by $\frac{1}{3}$, and the quotient will bee $\frac{3}{2}$, that is $1\frac{1}{2}$, whereby I perceyue that the proportion in this question is, as 3 to 2. Therefore (as you taught mee euen nowe) I multiply 3 by 2, and the summe is 6, which must bee the middle number: then I multiply the middle number 6 by 2, which is the least Terme, and the summe is 12, that we I diuide by 3, beyng the greater Terme, and the quotient is 4, so is 4 the leaste number of the 3. Then I multiply 6 by 3, whereof commeth 18, and that I diuide by 2, and so haue I 9, which is the greatest number of the 3.

betweene twoo numbers,

Mayster. An other way yet may you finde the thirde number in any progression, if you haue 2 of them: for if the middle number bee one of them which you haue, then multiplye it by it selfe (as in this example 6 by 6 maketh 36) and that totall diuide by the other number which you haue, and the thirde number will be the quotient.

Scholer. Then if I diuide 36 (which cōmeth of 6 multiplyed by it selfe) by 4, the quotient will bee 9: and if I diuide 36 by 9, the quotient will be 4. But what if I knowe the firste number & the thirde, and woulde haue the middle

middle number?

Mayster. Multiply the 2 numbers togither, and in their totall you muste seeke yͤ roote of that number, & it shall bee the middle number: but bycause as yet you haue not learned howe to extract rootes, therefore vse the firste forme which I haue taught you, tyll I teach you to extract rootes. And now go forewarde with the aunswere to the same question.

Scholer. I perceyue then that the sonne muste not haue $\frac{1}{2}$ of the goods, nother yͤ mother $\frac{1}{3}$, nor yet the daughter $\frac{1}{2}$, but yet muste the goods bee diuided in such proportion, that the sonne shall haue 9 crownes for 6 to his mother: and the mother shall haue 6 crownes for euery 4 to the daughter. Then I apply it to the Golden rule in 3 examples thus: where the firste number is the Addition of those three nūbers 9, 6, 4, and the thirde is one of them seuerally: the seconde is the totall of the goods in the testament: & then by the worke of the Golden rule I finde out yͤ fourth number in euery worke, that is for the

19 — 3600
9

19 — 3600
6

19 — 3600
4

sonne

ſonne 1705 $\frac{5}{11}$: for the mother
1136 $\frac{16}{19}$: and for the daughter
757, $\frac{17}{19}$, whiche three ſummes
added togither do make the ſūme
of the whole goodes, as may bee
ſeene by this example.

1705	$\frac{5}{19}$
1136	$\frac{16}{19}$
757	$\frac{17}{19}$
3600	

And this (me thinketh) I doe perceyue, that bicauſe in this caſe there is a neceſſary remedy diuided agaynſte an vrgent inconuenience, therfore thoſe learned mē thought they might vſe the like liberty in that other queſtion.

Mayſter. Your geſſe is good, but they had ſo good reaſon for them in ye one, as they haue in the other: as in an other example of theirs it may better appeare, that is this.

A man left vnto his iij. ſonnes 7851 crounes, to bee parted in this ſorte, that the firſte ſonne ſhould haue $\frac{1}{2}$, the ſeconde ſonne $\frac{1}{3}$, and the thirde ſonne $\frac{1}{4}$, whiche is not poſſible, for $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ dothe make $\frac{26}{24}$ or $\frac{13}{12}$, that is $1\frac{1}{12}$, ſo is it more than the whole: but reduce theſe fractions into one denomination, and they will be $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, and ſo may you parte the goodes in ſuche proportion as theſe 3 Numeratours beare togither: that is, the firſte to haue 6 for euery 4 to the ſeconde: and the ſeconde to haue 4 as often as the thirde hath 3: and ſo theyr

Another queſtion of a Teſtamēt

portions will bee for the first, $3623\frac{7}{13}$: for the seconde $2415\frac{9}{13}$: and for the thirde $1811\frac{10}{13}$, and those 3 shares added togither, will make ye totall summe of the whole goodes, as you may easily see in this example.

$$\begin{array}{r} 3623\frac{7}{13} \\ 2415\frac{9}{13} \\ 1811\frac{10}{13} \\ \hline 7851 \end{array}$$

An other question is there proponed thus.

An other like question.

There is 450 Crownes to bee diuided betweene 3 men, so that the firste man must haue $\frac{1}{2}\ \frac{1}{3}$, the second man $\frac{1}{3}\ \frac{1}{4}$, the thirde man shall haue $\frac{1}{4}\ \frac{1}{5}$.

Scholer. I maruaile that any man should bee so ouerseene to propounde that question as a thing possible, sithe $\frac{1}{2}\ \frac{1}{3}$, $\frac{1}{3}\ \frac{1}{4}$, $\frac{1}{4}\ \frac{1}{5}$, doe make $1\ \frac{1}{3}\ \frac{1}{5}$, that is almost double the whole summe.

But I perceyue it might be thus proponed, that as often as the firste man did receyue 50 Crownes, so often the second man should receyue 35, and the thirde man 27, for $\frac{1}{2}\ \frac{1}{3}$, is equall to $\frac{50}{60}$, and so is $\frac{1}{3}\ \frac{1}{4}$ equall to $\frac{35}{60}$, and $\frac{1}{4}\ \frac{1}{5}$ is $\frac{27}{60}$, and so working the question, the 3 figures will appeare in this forme: whereby the firste mannes portion is founde to bee $200\frac{50}{56}$: the seconde mannes parte

$$\begin{array}{lcl} 112 & \diagdown\!\!\diagup & 450 \\ 59 & & 200\frac{50}{56} \end{array}$$

is

is 140 $\frac{35}{56}$: the thirde mans ſhare is 108 $\frac{24}{76}$: which in yͤ whole doth make 450 crounes that was yͤ whole ſumme to bee diuided betweene them.

112 — 450
35 — 140 $\frac{35}{56}$

112 — 450
27 — 180 $\frac{27}{50}$

Mayſter. And thus you are (I thinke) ſufficiently inſtructed in the rule of felowſhip.

THE RVLE OF ALLIGATION.

Now will I go in hande with the rule of Alligation, whiche hath his name, for that by it there are dyuers parcels of ſundry pryces, and ſundrie quantities alligate, bounde or mixed togither, whereby alſo it might be well called the rule of mixture, and it hath great vſe in compoſition of medicines, and alſo in mixtures of metales, and ſome vſe it hathe in medicines of wines, but I wiſhe it were leſſe vſed therein than it is now a dayes. The order of the rule is this: When any ſummes are proponed to be mixed, ſet them in order one ouer an other, and the common number whereunto you will reduce

The rule of mixture.

The reaſon of this rule.

 them,

them, set on the left hande, then marke what summes be lesser than that common number, & whiche be greater, and with a draught of your penne, euermore linke ij. numbers togither, so that one be lesser than the cōmon number, and the other greater than hee, for two greater or two smaller can not well bee linked togither, and the reason is this, that one greater and one smaller may bee so mixed, that they will make the meane or common number very well: but ij. lesser can neuer make so many as the common number, beeing taken orderly: no more can two summes greater than the meane, neuer make the meane in due order, as it shal appeare better to you hereafter. And as it is of necessitie to linke euerye smaller (once at the leaste) with one greater, and euerye greater with one smaller, so it is at libertie to linke them oftner than once, and so may there bee to one question many solutions. When you haue so linked them, then marke howe much eche of the lesser numbers is smaller than the meane or common number, and that difference set agaynst the greater numbers which be linked with those smaller, eche with his match still on the right hande, and likewise the excesse of the greater numbers aboue the meane, you

shall

ſhall ſet before the leſſer numbers whiche bee combined with them. Then ſhall you by Addition bring all theſe differences into one ſumme, which ſhall bee the firſt number in the Golden rule: and the ſeconde number ſhall be the whole maſſe that you will haue of all thoſe particulars: the thirde ſumme ſhal be eche difference by it ſelfe, and then by them ſhall bee founde the fourth number, declaring the iuſt portion of euery particular in that mixture. As nowe by theſe examples I will make it playne.

There is foure ſorts of wine of ſeuerall prices, one of 6 d, a galon, an other of 8 d, the thirde of 11 d, and the fourth of 15 pence the galon, of al theſe wines would I haue a myxture made to the ſumme of fiftie galons, and ſo that the price of eche galon may be ix. pence. Nowe demaunde I howe much muſt bee taken of euery ſort of wine? — A queſtion of mixing of vvine.

Scholer. If it ſhall pleaſe you to worke the firſt example, that I maye marke the applying of it to the rule, then I truſt I ſhall be able not onely to do the like, but alſo to ſee reaſon in the order of the worke.

Mayſter. Marke then this forme, and the placing of euery kinde of number in it.

	The prices seuerall.	The diffe- rences.	
The com- mon price.	6	6	a
	8	2	b
☞ 9	11	1	c
	15	3	d
		12	

a
12 — 50
6 — 25

b
12 — 50
2 — $8\frac{2}{6}$

c
12 — 50
3 — $4\frac{1}{6}$

d
12 — 50
3 — $12\frac{3}{6}$

Heere (you see) I haue set downe the seuerall pryses whiche bee 6, 8, 11, 15, and haue lynked togither 6 with 15, and 8 with 11. The common price 9, I haue set on the lefte side: And the difference betwene it, and euery particular pryce, I haue sette on the righte hande not agaynst the summe, whose difference it is, but agaynst the summe that it is linked withall: so the difference of 15 aboue 9, is 6, which I haue set not agaynst 15, but agaynst 6, that is linked with 15, and the difference betweene 6 and 9 (that is 3) I haue set agaynst 15. So lykewayes the difference betweene 8 and 9 is but 1, that haue I set agaynste 11, and the diffenrence of 11 aboue 9 (whiche is 2) I haue set agaynste 8. Then adde I all those 4 differences, and they make 12, whiche I sette for the firste number in the Golden rule: the seconde number I make 50, which is the summe

of

of the gallonnes that I woulde haue, and the thirde ſumme is euery particular diffe=rence. Nowe if you worke by the Golden rule, you ſhall finde the number of gallons that ſhall bee taken of eche ſorte of wyne: For the better diſtinction whereof, I haue ſette theſe letters, a, b, c, d, bothe agaynſte the num=bers for whiche the woorkes doe ſerue, and o=uer the woorkes alſo, whiche ſeuerally ſerue for eche of them. And nowe if you liſte to ex=amine the truthe of theſe worrkes, adde thoſe 4 ſummes togither, and they wyll make 50, that is the totall whiche I woulde haue, as by this example you may eaſilye perceyue. And for to proue how the prices doe agree, do this. Multiplye this totall ſumme 50, by the common price 9, and it will make 450: then keepe that ſumme by it ſelfe, and afterwarde Multiplye euerye ſeuerall ſumme of Gallons, by the pryce belonging to the ſame Gallons, and if that ſumme doe agree with this, whiche you haue kepte firſte, then is your woorke well done. As here, 25 is the number of gallons of 6 d price, mul=

The proofe of this rule.

$$\begin{array}{r} 25 \\ 8\frac{2}{6} \\ 4\frac{1}{6} \\ 12\frac{3}{6} \\ \hline 50 \end{array}$$

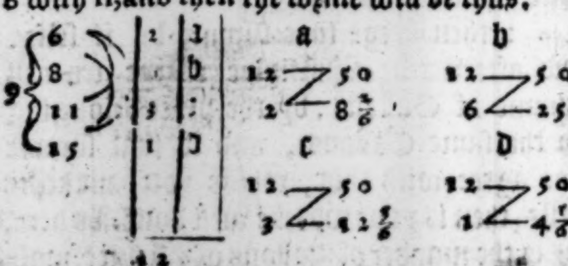
 tiply

tiplye then 25 by 6, and it maketh 150, which you shall set downe: then multiply $8\frac{2}{8}$ by 8, which is the price for that number of galons, and it will make $66\frac{4}{8}$: so agayne $4\frac{1}{8}$ multiplyed by 11, dothe make $45\frac{5}{8}$. And laste of all $12\frac{3}{8}$ multiplyed by 15, maketh $187\frac{3}{8}$. And these added togither dothe make 450, as in ye example annexed you may see: wherfore seeing it doth agree wt the former summe of 50, multiplyed by 9, I may iustely affirme this worke to be good and well done.

150
$66\frac{4}{8}$
$45\frac{5}{8}$
$187\frac{3}{8}$
450

And now to proue how you can doe ye like, I propounde the same question, onely willing you to vse some other forme of combining or linking the summes.

Scholer. That shall I proue with your fauoure, and therefore I combine 8 with 15, and 6 with 11, and then the forme will be thus.



whereby

whereby amounteth the same summe in totall of the differences, as did before: and yet now the differences be altered, as the combination is chaunged, whereof I vnderstande the reason by youre former worke. And therefore heere appeareth no straunge thing, but that nowe I muste haue $8\frac{2}{6}$ galons, of 6 pens, and 25 galons of 8 d. and 12 galons and $\frac{3}{6}$ of 11 d, and so consequently 4 galons and $\frac{1}{6}$ of 15 d, so that multiplying $8\frac{2}{6}$ by 6, it maketh 50, and then 25 multiplyed by 8, maketh 100: likewayes $12\frac{3}{6}$ multiplyed by 11, yeldes $137\frac{3}{6}$, and $4\frac{1}{6}$ multiplyed by 15, maketh $62\frac{3}{6}$, which 4 summes added in one, will yelde in the totall 450, which agreeth with the Multiplication of 50 (being the totall summe of galons) by 9 the common or meane price.

50
200
$137\frac{3}{6}$
$62\frac{3}{6}$
450

Mayster. Seing you conceyue this worke so well, I will propound an other example vnto you of more varietie in the Alligations or combinings: As thus.

A Marchaunte being minded to make a bargayne for spices in a mixte masse, that is to saye, of Cloues, Nutmegges, saffron, pepper

A questio of spices.

Ginger, and Almondes, the cloues being at 6 ſ. a pounde, the Nutmegges at 8 ſ. Saffron at 10 ſ. Pepper at 3 ſ. Ginger at 2 ſ. and Almondes at 1 ß.

Now woulde hee haue of eche sorte some, to the value of 300 ℔, in the whole, and eche pound one with an other to beare in price 5 ß. howe muche shall hee haue of eche sorte?

Scholer. That will I trye thus.

First I set downe those sixe seuerall prices, and at the lefte hande

5 { 1, 2, 3, 6, 8, 10 | 5 a, 1 b, 3 c, 3 d, 2 e, 4 f | 18

a: 18 — 300; 5 — $83\frac{1}{2}$

b: 18 — 300; 1 — $16\frac{12}{18}$

c: 18 — 300; 3 — 50

d: 18 — 300; 3 — 50

e: 18 — 300; 2 — $33\frac{1}{3}$

f: 18 — 300; 4 — $66\frac{2}{3}$

I set the common price 5 ß. Then I lynke them thus, 1 with 10, 2 with 6, and 3 with 8.

Maister. I had minded to haue combined them in more varietye, but I am content to see youre owne worke firste, and then more varieties in combination may followe anone.

Scholer.

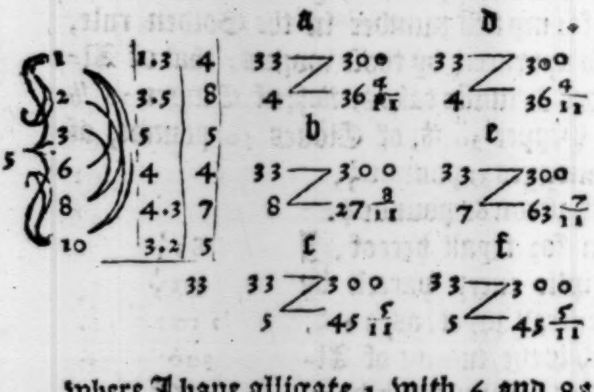
Scholer. Then to continue as I beganne, I seeke the difference betweene 1 and 5 (which is 4) and that I set agaynst 10: then agaynste 1 I set 5, whiche is the excesse of 10 aboue 5: so I gather the difference betweene 2 and 5, which is 3, and that I set agaynst 6, bycause it is combined with 2: and likewayes the difference of 6 aboue 5 (whiche is 1) I set agaynst 2. Then take I the difference of 3 from 5, which is 2, and that I set agaynst 8, and before that 3 I set the difference of 8 aboue 5, whiche is 3. Then gather I all these differences by Addition, and they make 18, whiche I set for my first number in the Golden rule, and so appeareth by those workes, that of Almondes I muste take 83 ℔ $\frac{1}{3}$, of Ginger 16 ℔ $\frac{2}{3}$, of Pepper 50 ℔, of Cloues 50 pounde, of Nutmegges 33 pounde $\frac{1}{3}$, & of Saffron 66 pounde $\frac{1}{3}$.

Then for tryall hereof, I	83 $\frac{1}{3}$
multiplie euery parcell by	33 $\frac{2}{3}$
his seuerall pryce, as 83 $\frac{1}{3}$,	150
which is the summe of Al-	300
mondes, I multiplie by 1,	266 $\frac{2}{3}$
which is their price.	666 $\frac{2}{3}$
Also 16 $\frac{2}{3}$ the summe of	1509
Gynger I multiplie by 2,	

which

which is the price of it. And so eche other in his kinde, as this table annexed dothe represente: and then adding them all togither, I finde the totall to be 1500, whiche also will amounte by the multiplication of the grosse masse of 300 by the common price 5, wherefore it appeareth well wrought.

Mayster. Nowe will I make the Alligation to proue youre cunning somewhat better, but bycause you shall not thinke youre selfe pressed to muche, I will also note the differences, as in this example you may see.



where I haue alligate 1 with 6 and 8: and therefore haue I set agaynst 1, bothe their differences: that is 1 and 3. Likewayes bycause 2 is combined with 8 and 10, I set before him their

their differences, 3 and 5. Agaynst 3 I haue set onely 5, whiche is the difference of 10, with whome 3 is combined onelye: lykewayes 6 is onelye Alligate to 1, and therefore is the difference of 1 onely set agaynste it: 8 is linked with 1 and 2, and therefore hath hee agaynst him bothe their differences, 4 and 3: and 10 is ioyned with 2 and 3, therefore hath he their differences 3 and 2. And bicause of ease for you, in an other columme I haue set the differences reduced into one number, for euery seuerall sorte, and haue also added them togither, whereby appeareth that they make 33, and so consequently you see the workes of the Golden rule set foorth for the sixe seuerall drugges: I haue added letters, a, b, c. &c. as before. But I would not wishe you to cleaue still to these elementarie aydes, but accustome memorie to trust to hir selfe, so shall occasion of negligence bee best auoyded. And as for the proofe, trie it at a more leysure, bycause ye time nowe is shorte, and you sufficientlye instructed in that proofe. And there resteth dyuerse things behinde yet, of which I woulde gladly giue you some taste before our departure.

Scholer. But if it may please you to lette me see all ye variations of this question, before

you

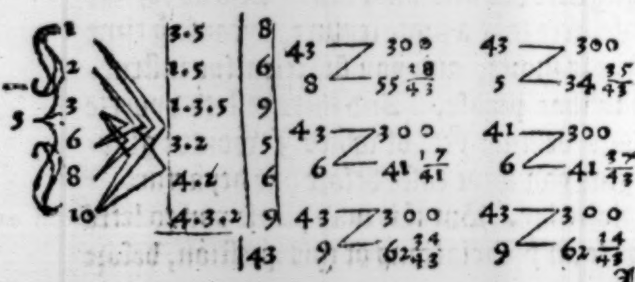
you go from it, for me thinketh I could varie it two or three wayes more yet.

Mayster. I am content to see you make two or three variations, but I woulde be loth to stay to see all the variations, for it may bee varied aboue 300 wayes although manye of them would not well serue to this purpose.

Scholer. I thought it impossible to make so many variations.

Mayster. Marvaile not thereat, for some questions of this rule may bee varied aboue 1000 wayes, but I would haue you forget such fantasies, till a time of more leysure. And now go forwarde with some variation of this question.

Scholer. For the first variation, I linke the firste number 1 with 8 and 10, and 2 I combine with 6 and 10, then ioyne I 3 with 6, 8. and 10, as in this forme.

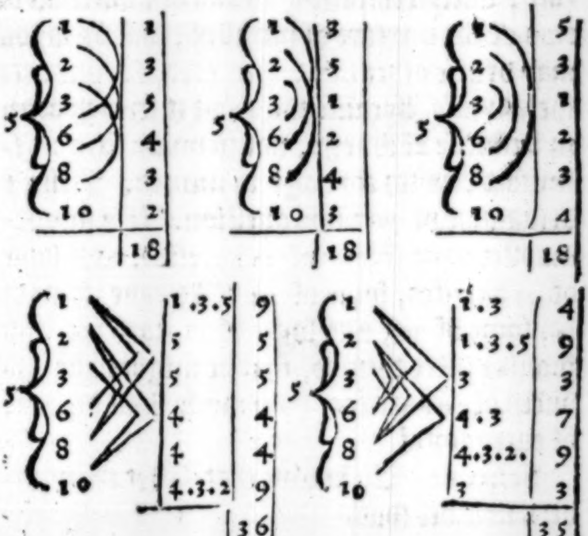


And

And so dothe there appeare the portion of whight for euery kinde of drugge in this mixture. Now for the triall.

Ma. Nay staye there, you shall not neede to make triall in one exãple so often, or if you list to doe it by your selfe, I am content. But nowe set forth (for declaration that you conceyue the rule) two or three examples of seuerall combinations, and then will wee passe to some other example, and so ende this rule.

Scholer. As it pleaseth you so will I doe. And these bee the varieties in which as the



combi-

combinations are seuerall, so dothe it playnely appeare, that the differences by which the proportion of ech seuerall kinde is taken, are also seuerall. And yet I see in the three first of these v. varieties, and in one other before the totall summe of the differences to be one, that is to say 18, whereby I perceyue that the varietie of their mixture dothe depende of the varietie of their differences seuerall, and not of the varietie of their totall summe.

Mayster. So is it. And seeing you conceyue it so well, I will make an ende of this rule, onely exhibiting to you one question or two of the mixture of metalles, that by it you may deuise other like, and exercise your selfe therein also, bycause the vse of it serueth often in businesse of charge, not so much for goldesmithes, as for coynage in mintes. Firste I demaunde of you this question. If a myntemayster haue Golde of 22 karectes, and some of 23 karectes, some of 24: Agayne some of 15, some of 16, and some of 18 karectes, and woulde mixe them so, that he might haue 100 vnces of 20 karectes, how much shall hee take of euery sorte?

A question of mixing of Golde.

Scholer. To knowe that, I set the numbers in order thus.

20	15	2
	16	3
	18	4
	22	5
	23	4
	24	2
		20

20 — 100   2 — 10   20 — 100   5 — 25

20 — 100   3 — 15   20 — 100   4 — 20

20 — 100   4 — 20   20 — 100   2 — 10

Mayster. You haue wrought the question well, but how chaunced you made no doubt of that new name, Karecte?

Scholer. Bycause I thought it out of tyme to demaunde such questions now, seeing you make so much haste to ende: and agayne in this case the proportion of the numbers is sufficiēt for my purpose in this worke, trusting that an other tyme you will instruct mee as well of this, as of sundry other things, which I haue hearde you talke of, so I haue a greate desire to know them.

Mayster. Your answere is reasonable: and your request and trust with Gods helpe I intende to satisfie. And nowe to go forwarde with this matter, let me see your examination of this laste worke,

Sch. First for the one parte I adde togither

all the particular summes as they ap-
peare in ẏ worke, and they make 100,
as heere by their addition it doth ap-
peare.

10
15
20
25
20
10
100

And so it seemeth that the summes are well gathered, but for the farther tryall of them, I multiply firste 20, which is the common or meane summe of the karectes by 100, which is the summe of the whole masse which I would haue, & it maketh 2000. Then I multiply euery particular sūme by the karectes that it dothe contayne, as 10 by 15, and that maketh 150.

150
240
360
550
460
240
2000

Likewise I multiplye 15 by 16, and it yeldeth 240: so 20 by 18 makith 360. And 25 by 22 yeldeth 550: likewayes 20 by 23 bringeth forthe 460: and last of all 10 multiplyed by 24, yeldeth 240: which summes all ioyned togyther make 2000, that dothe agree with the like summe before: wherefore I maye well saye that the worke is good. And nowe if it please you I woulde set forth some varieties of this question, to proue my witte.

Maister. Go to, let me see.

Scholer. Heere bee foure varieties.

20	15	3.4	7
	16	3	3
	18	2	2
	22	2	2
	23	5.4	9
	24	5	5
			28

20	15	2.3	5
	16	3.4	7
	18	4	4
	22	5	5
	23	5.4	9
	24	4.2	6
			36

20	15	2.3.4	9
	16	4	4
	18	3	3
	22	5	5
	23	5.2	7
	24	5.4	9
			37

20	15	4	4
	16	4	4
	18	2.3.4	9
	22	2	2
	23	2	2
	24	5.4.2	11
			32

And more yet I coulde make, but not lyke to the number that you spake of in the variation of the other question.

Mayster. That will I teach you at more leysure, seeing it is a thing rather of pleasure, than of any necessitie.

But nowe for youre exercise in this rule, one other question I will propose. A minte mayster hathe 6 ingottes of siluer of sundrie finenesse, some of 4 unces fine, and some of 5 unces, some of 6, and other of 8, some of 11, and other of 12: and his desire is to mixe 500

pound weight, so that in the whole masse euery pounde weight should beare 9 vnces of fine siluer, howe muche shall hee take (saye you) of euery sort of syluer?

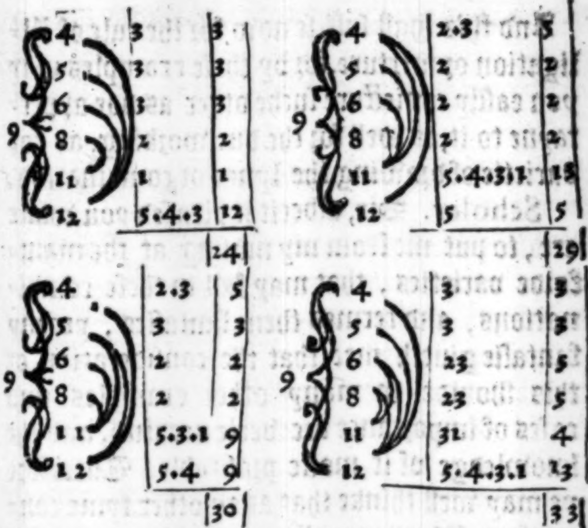
Sc. To finde out that I set the numbers thus in order.

9	4	2	2
	5	2	2
	6	3	3
	8	3	3
	11	5.4	9
	12	3.1	4
			23

And gathering the differẽces, it will appeare, that of the first sorte there must be $43\frac{11}{23}$: of the seconde like muche: of the thirde sorte, $65\frac{5}{23}$: and of the fourth sorte as much: of the fifth sorte $195\frac{15}{23}$: and of the sixth sorte $86\frac{22}{23}$, which in the whole will make 500 ℔ waight: and in vnces 4500, that is of the first sorte $173\frac{21}{23}$: and of the seconde sorte $217\frac{9}{23}$: of the thirde sorte $391\frac{7}{23}$: of the fourth sorte $521\frac{17}{23}$: of the fifth sorte $2152\frac{4}{23}$, and of the sixth sorte $1043\frac{11}{23}$, whiche all togither doe make 4500 vnces, agreeable to the multiplication of 9 by 500.

Mayster. This is well done of you, therefore nowe make three or foure varieties, and so an ende of this rule.

Sc. These 4 varieties I set for example.



Mayster. And by these it appeareth that you can find out more, with which I will not now meddle, saue onely for to shew you an easie helpe in drawing the lines of Combination, I will set forth two varieties heere.

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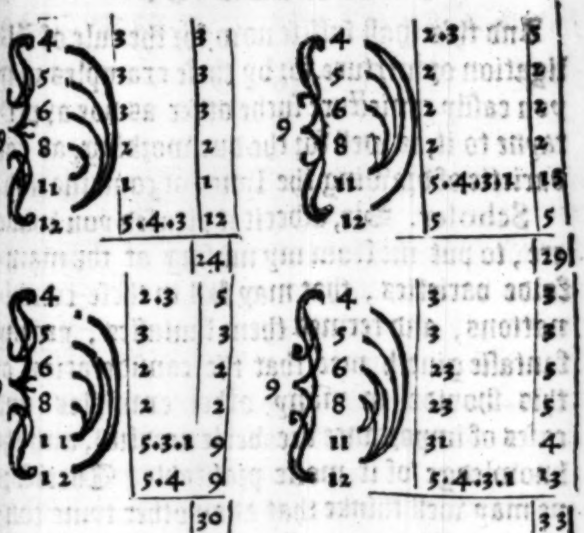

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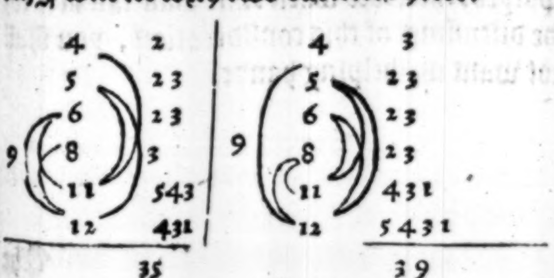


And

And this shall suffise now for the rule of Alligation or mixture, for by these examples may you easily coniecture suche other as doe appertayne to it, as well for the due working, as for varietie of drawing the lynes of combination.

Scholer. Sir, albeit it pleased you while ere, to put me from my musing at the manifolde varieties, that may fall in these combinations, and termed them fantasies, yet my fantasie giueth mee that the consideration of this shoulde in many other examples and cases of importance bee verie needfull, and the knowledge of it moste profitable. Therefore ye may well thinke that at an other tyme conuenient I will request you to ayde mee heerein.

Mayster. Truthe it is, that this consideration may fall in practise as well politike, as philosophicall, and sundrie wayes in them bee applyed, therefore when time shall fall fitte for the discussing of this consideration, you shall not want my helping hande.

THE RVLE OF FALSEHODE.

NOwe will I brieflye also teache you somewhat of the rule of Falsehode, which beareth his name, not for that it teacheth any fraude or Falsehode, but for that by false numbers taken at all aduentures, it teacheth howe to finde those true numbers that you seeke for. The occasion of the name.

Scholer. So might any other rule bee called the rule of Falsehode, for they worke by wrong numbers, and by them finde out the right numbers, so dothe the rule of Alligation, the rule of Fellowship, and the Golden rule partely.

Mayster. In the Golden rule, the rule of Felowship, & the rule of Alligation, although the numbers that you worke by, bee not the true numbers that you seeke for, yet are they numbers in iuste proportion, and are founde by orderly worke: where as in this rule, the numbers are not taken in any proportion, nor founde by orderly worke, but taken at all

aduentures.

And therefore I sometimes beeing merrye with my friendes, and talking of such questions, haue caused them that proponed such questions to call vnto them such children or ideotes, as happened to bee in the place, and to take their answere, declaring that I woulde make them to solue those questions that semed so doubtfull.

And in deede I did answere to the question and worke the triall thereof also, by those answeres which they happened at all aduentures to make: which numbers seeing they be taken as manifest false, therefore is this rule called the rule of False positions, and for briefnesse, the rule of Falshode, which rule for readinesse of remembraunce, I haue comprised in these fewe Verses following, in forme of an obscure Riddle.

Gesse at this worke as happe doth leade,
By chaunce to truthe you may procede.
And firste woorke by the question,
Although no truthe therein be done.
Suche falsehode is so good a grounde,
That truthe by it wyll soone bee founde.
From many bate to many mo,

From

From to fewe take to fewe also.
With to muche ioyne to few agayne,
To to fewe adde to many plaine.
In crosse waies multiply contrary kinde,
All truthe by falsehode for to finde.

The sense of these Verses, and the summe of this rule, is this.

When any question is proponed appertayning to this rule, firste imagine any number that you lyste, which you shall name the Fyrste position, and put it in steede of the true number, and then woorke with it as the question importeth: and if you haue missed, then is the laste number of that woorke either to greate or to little: that shall you note as hereafter shall bee taught you, and you shall call it the Firste errour.

Then beginne agayne, and take an other number which shall be called the Seconde position, and woorke by the question: if you haue missed agayne, note the excesse or defaulte as it is, and call that the Seconde errour. Then multiplye crosse wayes the Fyrste position by the Seconde erroure, and agayne the Seconde position by the Firste erroure, and note their totalles seuerally by the names of Totalles.

Then marke whether the two erroures were bothe like, that is to saye, bothe to muche, or bothe to little: or whether they be vnlike, that is, the one to muche, and the other to little, for if they bee like, then shall you subtract the one totall from the other (I meane the lesser from the greater) and the Remainer shall bee your diuidende, so muste you abate the lesser errour out of the greater, & the residue shall be the diuisour. Nowe diuide the diuidende by that diuisour, and the quotient will shewe you the true number that you seeke for. But and if the errours bee vnlike, then muste you adde bothe those totals (which you noted) togither, and take that whole number for the diuidend, so shall you adde bothe the errours togither, and that whole number shall bee the diuisour, and the quotient of that Diuision shall gyue you the true number that the question seeketh for: and this is the whole rule.

Scholer. This rule seemeth so vnlike any other, that without some example I shall not easily vnderstande it.

A question of Masondrye.

Mayster. Therefore take this example: A Mason was bounde to buylde a wall in 40 dayes, and it was couenanted so with him, that euery day ye hee wrought, hee should haue

for

for his wages 2 ß, 1 pennie, and euery day that he wrought not, he shoulde bee amerced 2 shillings 6 d, so that when the wall was made, and the reckening taken of the dayes that hee wroughte, and of the other that hee wroughte not, the Mason had cleerely but 5 ß, 5 d, for his worke. Now do I demaunde, how many dayes did hee worke of those 40, and howe many did he not worke?

Scholer. I praye you expresse the order of the worke, that I maye partly by imitation, and partly by comparing it with the rule, bee able againe to do the like.

Mayster. This order shall you keepe in the worke of this rule: first take some number (as you liste) at aduenture, as for example: I say hee played 12 dayes, and wrought 28 dayes. Nowe cast you the wages of euery day and see whether it will agree with the summe of 5 ß, 5 d.

Scholer. The 28 dayes that he wrought after 25 pence by the day, yeldeth 700 d. Then the 12 dayes that he wrought not, at 30 pence eche day, doth amount to 360 pens, which if I abate out of 700 pens, there resteth 340: but you say he had not so much.

Mayster. Hee had but 65 pens, and by this

this supposition hee shoulde haue had 340: therefore is this summe to muche by 275, which summe I muste set downe after this sorte as you see here, where first I haue made a crosse (commonly called Saint Andrewes crosse) and at the ouer corner on the lefte hande I haue sette the first position 12, and at the other corner vnder it, I haue set 275 which is the firste erroure, with this figure +, whiche betokeneth too much, as this lyne, —— playne without a crosse line, betokeneth too little. On the right hande of the crosse I haue lefte two lyke rowmes for the seconde position and his erroure. Therefore to prosecute the worke, I suppose hee played 16 dayes, and wrought 24.

12

275 +

Scholer. I was a while in doubte why you named the dayes of his working, seeyng they bee not set in the figure: and I doubted how you knew them, or else whether that you did suppose them at all aduentures, as you did yͤ dayes that he played: but now I gather, that seeing 40 dayes is the whole time limited, then the dayes that hee played beeing supposed, the rest of 40 muste needes bee the dayes

that

that hee wroughte, and therefore 28 followed 12 of neceſſitie, and 24 followeth 16 alſo of neceſſitie: but yet I ſcarce perceyue why you ſet not in the figure as well 28 as 12.

Mayſter. It forceth not whiche of them I take, ſo that in the ſeconde poſition I take the numbers of the ſame nature that is heere bothe of working dayes, or both of ydle dayes, but now examine you this ſecond poſition.

Scholer. If hee played 16 dayes, then abating 16 times 30 d. the ſumme will bee 480 d. And for 24 dayes that hee wrought, euery day yelding 25 d, the totall is 600 d: ſo that abating 480 out of 600, there reſteth 120: and as you ſay it ſhoulde bee but 65, therefore it is too muche by 55, that muſte bee ſet on the right hande of the figure at the nether part, and ouer it on the ſame ſide 16, which is the ſeconde poſition, thus.

12	16
2754	554

And as I gather by youre wordes, it were al one if I did ſet 28 in ſteede of 12, and 24 in ſteede of 16.

Ma. So were it. But this ſhall you marke, that of what nature ſo euer the two poſitions bee, of the ſame nature is the Quotient. Therefore when the poſitions in this queſtion

are

The profe of this rule

are 12 and 16, which bothe beeing numbers of the playing dayes, the quotient shall declare the true numbers of the playing dayes, where as if the positions had beene 28 and 24, whiche are supposed to bee the working dayes, then woulde the quotient declare the true number of the working dayes, and not of playing dayes as it will do nowe And therefore to continue the worke of this question, and to finde the true number of playing dayes, I muste multiplie crosse wayes the first position 12 by 55, that is the seconde errour, and the totall will be 660: then I multiplie 275 by 16, and it yeldeth 4400. Nowe bicause the errours are like, that is to say, both too much, I muste subtracte 660, out of 4400, and so remayneth 3740, which is the diuidend. Againe I must subtract the lesser errour 55 out of 275 that is the greater errour, and there will remayne 220, which shall be the deuisor, then diuiding 3740, by 220, the quotient will be 17. Wherefore I say nowe constantly, that 17 is the true number of dayes that the Mason played: and then it followeth, that hee wrought 23 dayes, and so is the question answered.

Nowe for the order of tryall of this worke there

there needeth none other proofe but onelye this, to worke with this number according to the question, and if it agree, then appeareth the number to be it that you would haue. As here nowe seeing hee wrought 23 dayes, and muste haue for euery daye 25 pens, ye whole summe commeth to 575. Then againe seing he played 17 dayes, and muste abate 30 pence for euery day, the whole summe of the abatement wyll bee 510: therefore I subtracte 510 out of 575, and there will remayne 65, which maketh 5 ß, 5 d, the cleere wages of the Mason for his worke, according to the question.

Scholer. Nowe I truste I vnderstande the worke and the rule so well (and the better by this proofe) that I can bee able to doe the like. And for a proofe I take the same question all saue the laste number, where I will suppose that hee had 10 ß, for his wages cleere. And now to gesse at the number of the dayes that hee wrought, I suppose firste that he wrought 20 dayes, then say I, if he wrought 20 dayes, his wages muste bee 500 d, then did hee playe other 20 dayes, for which muste bee abated 600 d, and then hee leeseth 1000 d. And so am I at a staye, for it is not like to your former worke.

Maister.

Maister. You should haue required of me some question, and not haue taken a question of youre owne fantasying, vntill you were more expert in this arte: for so might you as well happen on an impossible question as on a possible: but nowe to go forwarde, consider that this number is to litle by 220, seeing he shoulde gayne by your supposition 120 d, and in this position he leeseth 100, those both make 220, which you shall set downe for the firste errour with this signe —, betokening to little, as here in this forme folowing doth appeare. And now for the rest go forward your selfe once againe.

20

220 —

Scholer. As my erroure hath vttered my follye, so it hath procured me better vnderstanding. Nowe therefore considering this position not to solue the question, I take an other, supposing that hee wrought 30 dayes, then for his wages he must bee allowed 750 d, and for the 10 dayes which hee wrought not, hee muste abate 300 d, and so remayneth cleere 450 d: but it shoulde be onely 120 d, therefore is it to much by 330, which I set downe in the figure with the former position, and his errour and the figure appeareth thus.

Nowe

Now muſte I multiply in croſſe waies 220, by 30, & it will bee 6600. Then agayne I multiplye 330 by 20, and it will be alſo 6600. Wherefore if I ſhall ſubtract the one out of the other, there will remaine nothing to be the Diuidende.

20	30
220 −	330 +

Mayſter. In this you forget your ſelfe againe: for in as much as the ſignes of the erroures bee vnlike, therefore muſt you worke by Addition, adding togither thoſe two totalles to make the diuidende, and adding alſo the two erroures to make the diuiſour. And bycauſe you ſhall no more forget this part of the rule, take this briefe remembraunce:

Vnlike require Addition,
And like deſire Subtraction.

Scholer. You meane, that if the erroures haue like ſignes, then muſte the diuidende and the diuiſoure bee made by Subtraction, as is taught before: And if thoſe ſignes bee vnlike (as in this laſte example they be) then muſte I by Addition gather the Diuidend and the Diuiſour. Therefore muſte I adde 6600 to 6600, and it will bee 13200, which ſhalbe the diuidend,

dend. Then againe I adde 220 to 330, and it will bee 550, which muste bee the diuisour: wherefore diuiding 13200 by 550, the quotient will bee 24, whereby I knowe that the Mason wrought 24 dayes: and then it foloweth that hee played 16 dayes.

Maister. Examine your worke whether it bee agreable to the question or no.

Scholer. For 24 dayes worke, the wages must bee 600 d. and for 16 dayes which the Mason wrought not, there muste bee abated 480 d, and thē remaineth cleare to the Mason 120 pens, as the question importeth, wherefore it is euident, that 24 is the true number of the dayes that he wrought.

Maister. Although you seeme now to understande this worke, yet to acquainte your minde the better with ẏ new trade of this rule, I thinke it good to propone to you fiue or sixe examples more, before I make an ende of it.

Scholer. Sir I thanke you, that you doe so consider my commoditie and profit in knowledge, for undoubtedly it is practise and exercise that maketh men prompte and experte in euery kinde of knowledge.

Maister. You say well, so that they folow some certaine preceptes to gouerne and rule

their

their practise by, else may practise procure custome of errour, and a repugnance to exactnesse of knowledge, namelye as long as the errour is not plainely knowne to the vulgare sorte. But to returne to our worke. There is a seruaunt that hath bought of veluet and damaske for his mayster 40 yeardes, the veluet at 20 ß, a yearde, and the Damaske at 12 ß, and when he commeth home, his maister demaundeth of him howe much he hath bought of eche sorte: I cannot tell (sayth he) exactly, but this I know, that I payde for damask 48 ß. more than I payde for veluet, now must you gesse how many yeardes there is of ech sort. A question of vveres.

S. Although the gesse seemeth difficult, yet I will proue what I can do: for I remember your saying, that it forceth not howe fonde or false the gesse be, so it bee somewhat to the question, and not an answere of a contrarie matter.

Therefore first I imagine that hee bought 20 yeardes of Damaske, for whiche he shoulde paye after the former price 240 shillings: then must hee needes haue of veluet other 20 yeardes (to make vp the 40 yeards) and that would coste 400 ß. So that the totall of the price of the damaske is lesse than the sum payd for veluet 160 ß, and shoulde bee more by 48.

therefore the firste erroure is 208 to little. Then begin I agayne, and suppose he brought of Damaske 30 yeardes that coste 360 ß, then had he but 10 yeardes of Veluet, whiche coste 200 ß: and nowe the price of Damaske is greater than the price of the Veluet by 160 shillings, and shoulde bee but 48, therefore is the seconde erroure 112 to muche, whiche I sette in fourme of a figure as heere dothe appeare. Then doe I multiplie in crosse wayes. 208 by 30 & the summe will be 6240. Also I

20 30

280— 112+

multiplie 112 by 20, and there will amount 2240. And in as muche as the signes of the errours bee vnlyke, I knowe I muste worke by Addition, therefore adde I those two totalls togither, and they make 8480, whiche is the diuidende: then adde I also the two errours togither. 208 and 112, and they make 320, whiche is the deuisor. Wherefore diuiding 8480, by 320, the quotient will be. 26 $\frac{1}{2}$, whiche is the true summe of yeardes of Damaske that hee boughte: and in Veluet 13 yeardes $\frac{1}{2}$, and that appeared by examination, thus: 26 $\frac{1}{2}$ yeardes of Damaske at 12 ß.

the

the yearde, maketh 318 ß, then in Veluet hee had but 13 yeardes and ½, that coste 270 ß. at 20 ß. the yearde. Nowe Subtract 270 out of 318, and there will remaine 48, which is the number of shillings that the Damaske did cost more than the Veluet.

Mayster. Nowe shall you haue a question of an other kinde. A question of debt.

There are three men that do owe money to me, and I haue forgotten what the totall summe is, and what the particulars be.

Scholer. Why? then is it impossible to knowe the debt.

Mayster. Peace, yeare to hastie: there is more helpe in it than you yet see. I haue three seuerall notes, whereby it appeareth that I did confer their dets togither, and founde the debt of the first and the seconde to amount to 47 lb, the debt of the first man and the third did make 71 lb, & the seconde man his debt wt the third, did rise to 88 lb. Now can you tell what euery mā did owe, & what was ye whole totall?

Scholer. Nay in good fayth: but as I perceyue that it must be foūd by coniecture, so will I gesse at it, supposing yt the first man did owe 20 lb, and the seconde man 30, and the thirde.

Mayster. Nay staye there, for you are to

 farre

farre gone alreadie, you may not suppose a seuerall summe for euery man, for it is ynoughe to suppose one summe for the fyrste man, and let the other ryse as the question importeth. Therefore seeing you sette the fyrst man his debte to be 20 lb, the second man can not owe 30 lb, for the declaration is that their debtes odded togither, did make 47 lb, so muste the seconde man his debt be but. 27 lb. Now this second debt with ye thirde must make 88, therefore subtracte 27 out of 88, and there will remayne 61, as the thirde man his debte. Then sayeth the declaration, that the first and thirde mennes debtes do make 71: but by this supposition they make 81 yt is 10 to muche: whiche I muste set for the first errour. Nowe worke you the seconde position.

Sc. I suppose the firste mannes debte to bee 24 pounde, then must the seconde mans debte (by youre declaration) bee but 23 lb. seeyng both they doe make but 47 lb. Also the seconde man his debt with the thirde, doe make 88 lb, and the seconde man oweth but 23, therfore the thirde man must owe 65 lb. Nowe the thirde mans debte with the first, shoulde make by the declaration 71 lb, and they doe make 89 lb: that is 18 lb to muche, and that is the seconde

errour

errour, which I set downe with the first, and their position in this forme: & then doe I multiply in crosse wayes 20 by 18, and it is 360. Also 10 by 24 maketh 240.

20 24

10+ 18+

And bycause the signes of the errours be like, I must worke by subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the diuidende: then doe I subtract 10 out of 18 by the same reason, and so is the diuisour 8, whiche is founde 15 tymes in 120, therefore I say, that the first man did owe 15 lb, and then the second man must owe 32 lb, for those 2 we make 47 lb, and the thirde man his debt is 56, for so much remaineth if I bate 15 out of 71, or if I take 32 out of 88.

Mayster. For the thirde example take this easie question for the varietie in worke. Two men hauing seuerall summes which I knowe not, we thus talke togither: the firste sayeth to the seconde, If you giue me 2 ß of your money, then shall I haue 3 times so much money as you: the seconde answereth: It were more reason, that our summes were made equall, & so will it be, if you giue me 3 ß of your mony. Now gesse what eche of them had.

The third question.

 Scholer.

Note. Scholer. I imagine that the firste had 9 s.

Mayster. Consider euermore in your imaginations that you take a likely summe, as in this question take such a summe that hauing 2 added vnto it, may bee diuided into 3 partes euen.

Scholer. Why? I remember you sayde before, it forced not howe fondely so euer I gessed.

Mayster. As for the possibility of the solution it is truthe, but for easinesse in worke, the aptest numbers are moste conuenient.

Scholer. I thoght no lesse, and therefore I toke 9 as an apte number to be parted into 3: but I perceyue I shoulde haue considered the aptnesse of that partition after the Addition of 2 vnto it, and then 7 had ben more meeter.

Maister. That is truthe, and then shoulde the seconde man his summe be 5: for although he haue now but the thirde parte of 9, that is 3, yet you must remember that he lent the firste man 2, and so had he 5.

Scholer. Then to go forwarde: if the seconde man had 3 of the first man, then shoulde he haue 8, and the firste man but 4, so hath hee double to the first man: yet he saide in the question they should haue equall: wherfore it ap-
pearetb

peareth that he hath 4 to much. Therefore I note that error with his supposition, and gesse agayne that hee hath 10 s: whereunto I adde 2 shillinges borowed of the seconde man, and then hath hee 12 shillinges, so the seconde man hath remayning but fower, whereunto if I adde the 2 that hee lent to the firste man, so had hee but 6 ß at the beginning. Then take 3 shillings from the firste man, and giue to the seconde, then hath the firste man but 7, and the seconde hath 9, which are not equall, but there are 2 to many, wherefore I set downe bothe the positions wt their errors as heere you see, and multiply a crosse, so commeth

there 40 and 14: and bycause the signes bee like, I take 14 out of 40, and so resteth 26 to bee the diuidende, then likewayes I take 2 out of 4, and there resteth 2, by which I di=uide 26, and the quotient will bee 13, which is the summe that the firste man had. And so appeareth that two beyng added thereto, the summe will bee 15, so hath the seconde man now but 5, and before he had 7: then take three from the first, and put to his seuen, so haue eche

of them 10, and that is equall, as the question woulde.

The fourth example.

Mayster. For the fourth example take this question. One man saide to an other: I thinke you had this yeare two thousande lambes: so had I saide the other: but what with paying the tythe of them, and then three seuerall losses, they are much abated: for at one tyme I loste halfe as many as I haue now lefte: and at an other tyme the thirde parte of so many: and the thirde tyme $\frac{1}{4}$ so many. Nowe gesse you how many are lefte.

Scholer. Bicause heere is mention made of certaine partes, I must take a number that may haue all those partes: that is to say, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, which will bee 24, howe bee it 12 hath the same partes. Therefore firste I take 12 to bee the number that doth remaine, so hath hee loste 6, 4, and 3, that is 13: and in the whole 25, but it shoulde bee 2000.

Mayst. Ye are deceyued yet still: you haue forgotten the 10 parte, which muste bee defalked, that is 200, so there remayneth but 1800: and now go on agayne.

Sc. Then to finde the erroure, I take 25 out of 1800. and there remayneth 1775 to fewe, which I set for the firste erroure. Then for

For the seconde position I take 24, whose halfe is 12, the thirde parte 8, & the quarter 6, whereby riseth 50, which is to little by 1750, therefore I set downe bothe ỹ positions wyth their errours thus.

12	╳	24
1775—		1750—

And multiply in crosse wayes 1775 by 24, wherof cōmeth 42600. Also I multiplie 1750 by 12, and there resteth 21000. And bicause the signes are like, I do subtract the one from the other, & so remayneth the diuidend 21600: then doe I subtract 1750 oute of 1775, and there resteth 25: by whiche I diuide 21600, & the quotient is 864, wherof the halfe is 432, and the thirde parte is 288, ỹ quarter is 216, whiche all beyng added togither, wil make 1800. And if you adde thereto the tenth which was abated before, then wil the whole summe be 2000. And nowe dothe there come a question to my memorie which was demaunded of me, but I was not able to answere to it, and now me thinketh I could solue it.

864
432
288
216
1800

Mayster. Propowne your question.

Scholer. There is supposed a Lawe made, that A questio

f sheepe d tillage.

that (for furthering of tillage) euerie manne that doth keepe sheepe, shall for euery 10 sheepe eare and sowe one acre of grounde: and for his allowance in sheepe pasture, there is appoynted for euerye 4 sheepe 1 acre of pasture: Nowe is there a riche sheepemayster which hathe 7000 acres of grounde, and woulde gladlye keepe as many sheepe as hee might by that Statute, I demaunde howe many sheepe shall hee keepe?

Mayst. Answere to the question your selfe.

Scholer First I suppose hee maye keepe 500 sheepe, and for them hee shall haue in pasture after the rate of 4 sheepe to an acre, 125 acres, and in earable grounde 50 acres, that is 175 in all: but this erroure is to little by 6825. Therefore I gesse againe, that hee may keepe 1000 sheepe, that is in pasture 250 acres: and in tillage 100 acres, which maketh 350: that is to little by 6650.

500 1000

6825— 6650—

These bothe errours with their positions I sette downe as you see, and multiplye in crosse 6825 by 1000, and it maketh 6825000. Then I multiplie 6650 by 500, and it doth amount

amounte to 3325000, whiche summe I do subtract out of the firste, and there remayneth 3500000 as the diuidende. Also I doe subtract the lesser erroure out of the greater, and so remayneth 175, by which I deuide ẏ sayde diuidend, and the quotient will be 20000, so that I see, that by this rate he that hath 70000 acres of grounde, may kepe 20000 shepe: and thereby I coniecture, that many men may kepe so many shepe, for many men (as the common talke is) haue so many acres of grounde.

Ma. That talke is not likelye, for so much grounde is in compasse aboue 48 $\frac{3}{4}$ myles, but leaue this talke and returne to your questions, least your poynting be scarce well taken.

Scholer. In deede I doe remember, that the Egiptians did grudge so much agaynste shepherdes, till at length they smarted for it, and yet they were but small sheepemaisters to some men that be now, and the shepe are waxen so fierce now and so mightie, that none can withstande them but the Lion.

M. I perceiue you talke as you heare some other: but to the worke of your question, bothe this last question, and the nexte before might bee wrought without the seconde position, by the rule of proportion, as this. When in this question

Another vvay of vvorking.

question ye founde in the first errour, that for 500 sheepe, there muste bee 175 acres, then might you reduce it to the Golden rule, thus. If 175 acres will admit in allowance 500 sheepe, than 7000 will haue 20000. And so

175 ⟋ 500
7000 ⟍ 20000

by one position with the helpe of the Golden rule may you answer that question. Likewise for the question of lambes, when you hadde founde that 12 came of 25, you might haue set the figure thus as you see, and

25 ⟋ 12
1800 ⟍ 864

haue sayde: If 25 doe leaue but 12, what shall 180 leaue? and it would appeare to be 864.

Scholer. Sir I thanke you for this ayde, for it doth much shorten the worke of this rule.

Another way yet.

Mayster. Yet againe I will shewe you an other waye to answere to this last question without this rule of False position, and that by the rule of Felowshippe, for it appeareth in the proponing of the question, that 10 sheepe must haue in pasture 2 acres and $\frac{1}{2}$, and for them must there be eared but one acre: so it foloweth that for two acres eared, there must be fiue set to pasture. And if you put them bothe into one sum, they will make 7. Therfore loke what

what proportion 7 being this totall, doth beare to 5 and to 2, such proportion shall any totall in this question beare to the pasture grounde and the eared grounde.

Scholer. This serueth wonderous aptly. Therefore to proue it, I demaunde thys by the former supposition: If a man haue 300 acres, how much shall he leaue in pasture, and howe much shall hee turne to tillage? You say that as 7 is to 5, so shall 300 bee to the acres of pasture: and as 7 is to 2, so is 300 to the acres of tillage, whereof for bothe I haue set examples here folowing, wherby appeareth ye of pasture there shall be $214\frac{2}{7}$ acres, and of tillage $85\frac{5}{7}$, which both summes added together, doe make 300.

7 Z 5
300 Z $214\frac{2}{7}$

7 Z 2
300 Z $85\frac{5}{7}$

Mayster. Now take an other example: A man hath three siluer cuppes with one couer, ye couer wayeth 18 unces, the second cup wayeth euen halfe the waight of the firste and ye third. Now if the couer be put to the firste cuppe, they weye iuste as much as all the three cuppes doe wey: and the couer be ioyned with the seconde cuppe, they weye as much as the seconde twice and the third: & if the couer be put to the thirde cuppe, Another question.

cup, they will make twice as much as the first and the seconde cup. Nowe try you what was the iuste waight of euery cuppe.

Scholer. I we set the waight of the firste cup to be 9 vnces: then in as much as these two (that is to say, the couer and the first cup) we wey the waight of ye three cuppes, I see that the three cuppes muste wey 27 vnces, for so much is 18 and 9. Also bicause the firste and the third we wey duble so much as the secōd, therefore is it the thirde parte of that waight, that is 9, and then woulde it follow, that the third cup also should wey 9 vnces, but then the question sayeth, that the couer being ioyned to the seconde cup, they wey as much as the seconde twice and the thirde once, that should bee 27, and so it dothe: then being ioyned with the third cup, they shoulde wey twice as much as the firste and the seconde, that shoulde bee 36, and they wey but 27, so is that erroure 9 to little. Thē beginne I againe, and say, that the firste cup doth wey 12 vnces, whiche I ioyne with the couer, and they make 30 vnces: then seeing the seconde is $\frac{1}{3}$ of that waight, it muste needes wey 10 vnces, and the thirde muste weye 8 vnces, seeing the firste and the thirde muste wey 20 vnces. Now put I the couer to the

the seconde cuppe, and they weye 28 vnces, which should be euen so: then ioyne I the couer with the thirde cuppe, and so should it wey twise the firste & the seconde, that is 44 vnces, and they do weye but 26, that is 18 to little: those erroures with their positions I set downe, & multiply in

9 12

9— 18—

crosse waies 9 by 12, whereof commeth 108. Also 9 by 18, and that yeldeth 162: and in as much as the signes bee like, I abate the lesser out of the greater, & there doth remaine 54. Then doe I also abate the lesser erroure from the greater, and so remaineth 9, by which I diuide 54, and the quotient is 6: which I take for the true weight of the first cup: which beinge ioyned with the couer, muste weye as much as the three cuppes, so doe they weye but 24 vnces. Then seeing the seconde cuppe is the third part of that weight, for the other two cuppes (you say) must wey double his weight, the weight of the seconde cuppe is 8 vnces, and so the weight of the thirde muste be 10 vnces. Now put the couer to the second cuppe, and it will make 26 vnces: that muste bee the weighte of the seconde twice, and the thirde

 once,

once, that is twice 8, and once 10, and so is it. Agayne, put the couer to the third cuppe of 10 vnces, and they must weye twice as muche as the firste and the seconde, that is 28: and so is all agreeable.

Mayster. Then aunswere to this question.

A question of water.

There is a Cesterne with. iiij. cockes, containing 72 barrels of water, and if the greatest cocke bee opened, the water will auoyde cleane in. vj. houres: at the seconde cocke it will aske 8 houres: at the thirde cocke it will auoyde in no lesse than 9 houres: and at the smallest it will require 12 houres: Nowe I demaunde, in what space will it auoyd, all the cockes being set open?

Scholer. Firste I imagine that it will auoyde in 2 houres.

Mayster. Then muste there auoyde by the firste cocke $\frac{1}{3}$ of the water, that is 24 barrelles, and by the seconde cocke $\frac{1}{4}$, that is 18, and by the third cocke $\frac{2}{9}$ that is 16 barrels, and by the smallest cocke $\frac{1}{6}$, that is 12 barrels, all whiche summes put togither doe make 70, as by their addition it doth appeare, but it shoulde be 72, therfore the errour is 2 to fewe.

Scho-

Scholer. Then I begin agayne by youre fauoure, bicause I thinke I vnderstande the worke, & put three houres for the due time: so shall there

24
18
16
12
———
70

run out at the greatest cocke $\frac{1}{2}$, ẏ is 36 barrels, and at the second hole $\frac{3}{8}$, that is 27, and at the third cocke $\frac{1}{3}$ that is 24, and at ẏ smallest hole $\frac{1}{4}$, that is 18 barrels, whiche altogither we make 105, and should bee but 72, so is it to muche by 33, therefore we I set the errors in order of ẏ figure, with their possitions, & worke by multiplication, in crosse, sayinge: 2 times 3, is 6, & 2 times 33 maketh 66: and bicause the signes are vnlike, I muste adde those two totalls togither, whiche make 72: also I adde the two erroures, and they make 35, by whiche I diuide 72, and the quotient riseth 2 $\frac{2}{35}$, whereby I see, that all the cocks beyng set open, the water will auoide in 2 houres and $\frac{2}{35}$ of an houre.

2 3
2 — 33 +

Mayster. This exercise maketh you to grow expert in the rule. Therefore I will inure you somwhat more with a questiō or two.

There were two men that had bene parteners, and had in accmopt betweene them 300

duckets: whereof the one should haue for his parte 180, and the other 120: but in the parting of them they fell at variaunce, so that eche of thẽ catched as many as he could: yet afterwarde beyng reconciled, they agreed that hee which had gotten most parte of them, shoulde lay downe $\frac{3}{4}$ of them againe, and hee that had gotten leaste, shoulde laye downe $\frac{1}{3}$ of those which he had taken, & then parting them vnto two equall partes, eche man too haue halfe thereof, and so had they their iust portions as they ought: now I demaunde of you what eche of them had gotten by the scambling?

Scholer. I suppose he that had least, gatte 108 duckets, then the other had 192: wherefore in laying downe agayne of the 192, there was put downe $\frac{3}{4}$, that is 144, and so had he left but 48. Also of the 108 there was layde downe 36, that is $\frac{1}{3}$, and so he had lefte 72. Then I put togither 144, and 36, and it maketh 180, which I parte into two partes euen, and so commeth 90 to bee giuen to eche of them: which summe put to 72, maketh 162 and ioyned to 148, it maketh 38: and nowe I doubt how I shall go forwarde.

Note. Mayster. You neede not to take but one of them which you list, the greater or the smaller, for

for all commeth to one purpose : and so maye you compare it that you take to anye of the other summes, remembring that you make comparison to the same in the seconde worke: as for example of the first parte. If you compare 138 with the lesser summe due, that is 120, so is it 18 to much: and if you compare it with the greater summe, then is it 42 to little. Againe, if you compare 162 to the greater summe, the errour will bee 18, as it was in the other: but it will haue a contrarie signe: & if you compare it with the lesser summe it will bee 42 to muche: so that the errour bothe wayes is eyther 18 or 42: and as for the signes it little forceth, for in them is nothing considered here, but lykenesse and vnlikenesse, whiche in this case dothe neyther further nor hinder. But now go on with the worke.

Scholer. If it be so, then am I out of my greatest doubt. Then I ioyne that 90 (which I founde as the halfe of ẏ latter particion) vnto 48, whiche is left with ẏ one man, and so hathe he 138, which (I may say) is 18 to many for the least shoulde be but 120: that errour do I note, & then make a new position, supposing the one man to haue 204, and the other to haue 96, wherfore of ẏ 204 there must bee layde

downe 153, and so remayneth with him 51. Al-
so of the 96 there must be layde downe $\frac{1}{3}$, that
is 32, and so resteth with that man 64. Now
of the 153 and 32 I make one summe as 185,
whiche I must diuide into 2 equall partes, and
so eche man shall haue 92 $\frac{1}{2}$, whereunto if I
adde theire former portions reserued, then the
one shall haue 156 $\frac{1}{2}$, & the other hathe 145 $\frac{1}{2}$.
Wherefore I take the lesser summe nowe a-
gayne as I did before, that is 145 $\frac{1}{2}$, and finde
ỳ he hathe to many by 23 $\frac{1}{2}$, for he should haue
but 122, so haue I for my two positions ij.
errours, whiche I set downe
as here may be sene, eche errour vnder hys position, and then by the rule I we multiply in cros

108	96
18 +	23 $\frac{1}{2}$

wayes 108 by 23 $\frac{1}{2}$, and there ryseth 2538
whiche I note: then agayne, I multiplye 96 by
18, and thereof amounteth 1728. Nowe bi-
cause the signes are bothe like, that is bothe to
many, I muste worke by Subtraction, & so a-
bating 1728 out of 2538, there will reste for
the Diuidend 810: then for the Diuisor I sub-
tract 18 out of 23 $\frac{1}{2}$, and there remayneth 5 $\frac{1}{2}$,
by whiche I diuide 810, and the quotient will
bee

be $147\frac{3}{11}$, which is ẏ iust portion of him that had ẏ least sum. And if I do subtract it out of 300 being the total sum, then wil there remain $152\frac{8}{11}$ as the portion that the other did get.

Maister. For the proofe of this worke, you maye chose whether you will examine those numbres according to the forme of the question, or els work by other two positions for to finde the seconde number: and if those positions bring the same numbers that did amount by the first two positions, then doth eche worke confirme other.

Scholer. By your patience, I will proue bothe wayes, not onely to see their agreement, but also to accustome my mind to those workes: for I perceiue it is exercise that must be the chief engrauer of these rules in my memory.

Maister. You consider it well: then go to.

Scholer. Firste I will by two other positions trye to finde the portion of him whiche had moste.

Mayster. Although you may do it with any positions, yet to see the agreement of your worke the better, take the same positions that you did before, comparing them nowe to the greater, as you did before vnto the lesser.

Scholer. Then I suppose that he that had moste, had 192, so had the other 108. Nowe if I take $\frac{3}{4}$ of 192, that will bee 144, and there will rest to that man but 48. And from the second whiche had 108, if I take $\frac{1}{3}$, that is 36, there will remaine to him 72: thē ioyning 144 with 36, it will make 180, the halfe whereof being 90. If I adde to eche of those two mens portions remaining with them, the one shall haue 138, and the other 162, of whiche two I take the greater (that is 162) and see it to be 18 to fewe, for it shoulde bee 180, that error I note vnder his position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then take I $\frac{3}{4}$ of 204, and it will bee 153, and there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is 32, and there remayneth to him 64. Nowe put I that 32 vnto 153, and it yeldeth 185: whiche being parted in equall halfes, maketh 92 $\frac{1}{2}$, to bee added to eche mannes remaynder, and so the one hath 143 $\frac{1}{2}$, and the other 156 $\frac{1}{2}$: wherefore I take the greatest summe, and it is 23 $\frac{1}{2}$ to litle, that doe I note also, and set both these errors vnder their positions, as in this example folowing doth appeare.

And

And then multiply-
ing 192 by $23\frac{1}{2}$, there
doth ariſe 4512.
Agayne, I multi-
ply 204 by 18, and

192		204
18 —		$23\frac{1}{2}$

it maketh 3672, whiche I doe ſubtract out of 4512, bicauſe the ſignes be like, and there reſteth 840 for the diuidend: then ſubtracting 18 out of $23\frac{1}{2}$, there will remayne $5\frac{1}{2}$, which I muſte take for the diuiſor. And ſo diuiding 840 by $5\frac{1}{2}$, the quotient will bee $152\frac{8}{11}$, whereby I haue found an agreable ſumme to that which I found by the former poſitions, for him that had moſte, which if I doe ſubtract out of 300, that is the totall, there will reſte $147\frac{3}{11}$, which was the portion of him that had the leaſte part.

Maiſter. So by diuers poſitions you ſee, that one doth confirme the worke of the other. Now examine thoſe two numbers by the forme of the queſtion, and ſo ſhall you proue your worke good alſo.

Scholer. If that hee which gat moſte, had $152\frac{8}{11}$, then muſte hee lay downe $\frac{3}{4}$ of his ſumme, that is $114\frac{6}{11}$, and ſo ſhall remayne with him but onely $38\frac{2}{11}$. The other which had leaſte, that is $147\frac{3}{11}$, muſte put downe

 of

of his summe $\frac{1}{3}$, that is $49\frac{1}{11}$, and so dothe there remayne with him yet $98\frac{2}{11}$. Then doe I adde together $114\frac{6}{11}$, and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$, whiche I muste parte into. ij. equall partes, and that will bee $81\frac{9}{11}$, to bee giuen to eche of them: so putting $81\frac{9}{11}$, vnto $38\frac{2}{11}$, there dothe amounte 120 iuste, whiche is the true portion of hym that should haue the lesser sum: and adding $81\frac{9}{11}$, to $98\frac{2}{11}$, the totall will be 180, the true portion of the other. And so is the worke by this proofe also tryed to be good. And this I marke by the way, that in their scamblyng, hee gat moste (as it chaunceth often) that ought to haue had least by iust partition.

Mayster. Let your studye bee to learne truth and iust arte of proportion, and to distribute and parte accordyng therevntoo, as often as occasion shal be ministred. And here would I make an ende of this rule, saue that I remember one pleasant question whiche I can not ouerpasse, which I will declare somwhat largely, bycause you shall as well vnderstande some reason in the pleasantinuention, as apte proceeding in the witty workyng therof.

[A]n example [o]f mixture Hiero kyng of the Syracusanes in Sicilia,

tillа, hadde caused to bee made a Crowne of Golde of a wonderfull weyght, to bee offered for his good successe in warres : in makinge whereof, the Goldesmith fraudulently toke out a certayne portion of Golde, and put in syluer for it, so that there was nothinge abated of the full wayght, althoughe there was muche of the valewe diminished. Whiche thinge at lengthe beynge uttered, (as no euill can alwayes lye hidde) the King was sore moued, and beynge desirous to trye the truthe without breakinge of the Crowne, proponed the doubte to Archimedes, unto whose witte nothinge seemed unpossible, whiche althoughe presently he coulde not aunswere unto, yet hee had good hope to deuise some pollicy for that inuention. And so musing thereon, as he chanced to enter into a bayne full of water to washe him, he obserued that as his bodye entred into the bayne, the water did runne ouer the tubbe: whereby his readye wit of suche small effectes coniecturing greater workes, cōceaued by and by a reason of solution to the Kings question, & therfore reioycing exceedingly more than if he had gotten the Crowne it selfe, forgat ẏ he was naked, & so ran home, cryinge as hee ran, εὕρηκα, εὕρηκα, I haue found, I haue founde.

of gold and syluer.

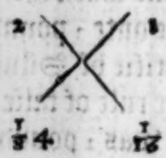
And

And therevppon caused ij. massie peeces, one of golde, and an other of siluer to bee prepared of the same weight that the sayde crowne was of: & considering that golde is heauier of nature than siluer, and therefore golde of lyke weight with Siluer, muste needes ocupie lesse roume, by reason it is more compacte and sounde in substance, hee was assured that putting the masse of Golde into a vessell brimme full of water, there woulde not so muche water runne ouer, as when hee shoulde put in the siluer masse of the like weight. Wherefore hee tryed bothe, and noted not onely the quantities of the water at eche tyme, but also the difference or excesse of the one aboue the other, whereby hee learned what proportion in quantitie is betweene golde and siluer of equall weight. And then putting the crowne it selfe into the vessell of water brimme full (as before) marked howe muche water did runne out then, and comparing it with the water ẏ ran out when the golde was put in, noted howe much it did exceede that: & likewayes comparing it to the water that ran out of the siluer, marked howe much it was lesse than that: and by those proportions founde out the iust quantitie of golde ẏ was taken out of the crowne, & howe

how much Syluer was put in steede of it. But seeyng Vitruuius which writeth this historie, dothe not declare the particular woorke of this tryall, it shall bee no inconuenience to suppose an example for declarations sake, wherein although the true and iuste propor- tions, be not expressed, yet the fourme of tryall shall bee truely set foorth. And for an exam- ple, I suppose the weight of the Crowne to be 8. ℔, and so of eche of ỹ other two masses. And when the masse of Golde was putte into the water, I imagine that there ran out 2 pounde of water: and when the masse of Syluer was put in, I suppose there ran out 3. pounde $\frac{1}{2}$. Againe when the Crowne was put in, there ranne oute 2 pounde $\frac{1}{4}$. Nowe to knowe what quantitie of Siluer was in ỹ Crowne, worke by the rule of false position, and imagine that there was 2 pounde of Syluer: then must there be 6 pounde of Golde. Then say thus by the rule of Proportion: If 8 pounde of Gold do ex- pell 2 ℔. of water, what shall 6 ℔. expell? and it will be 1 pounde $\frac{1}{2}$. Againe for the Syluer: If 8 ℔. of Syluer expell 3 ℔, $\frac{1}{2}$ of water, what shall 2 ℔ of Syluer put out? it will be $\frac{7}{8}$. Now adde those two weights of water togither, and they will make 2 ℔ $\frac{3}{8}$, and it shoulde be by the supposi-

suppoſition 2 li. $\frac{1}{4}$, ſo is it to muche by $\frac{1}{8}$.

Scholer. Now we I vnderſtande ẏ worke as I thinke, therfore I pray you let me worke the reſt of the queſtion. And bycauſe this firſte ſuppoſition did erre, I note that poſition and his errour, and take a newe poſition, eſteeminge the Syluer to be but one pound, ſo muſt there be in Golde 7 pounde, Then ſaye I: If 8 li of Golde yelde 2 ℔ of water, what ſhall 7 ℔ yelde? and it will be 1 ℔. $\frac{3}{4}$. Agayne if 8 li, of Siluer expel 3 ℔. $\frac{1}{2}$ of water, what ſhall 1 ℔, expell? and it will be $\frac{7}{16}$. Nowe muſt I adde thoſe two ſūmes togither, & they make 2 li, $\frac{1}{16}$, & they ſhould make 2 ℔ $\frac{1}{4}$, ſo is it to little by $\frac{1}{16}$ Therefore I ſette the poſitions with theire erroures in order, as heere foloweth. And thē I multiplie in croſſewayes 2 by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: lykewayes 1 multiplyed by $\frac{1}{8}$: maketh $\frac{1}{8}$.



And bicauſe the ſignes bee vnlike, I muſt adde thoſe ij. ſummes, whiche make $\frac{1}{4}$, and that is the diuidende. Agayne I muſt adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be $\frac{3}{16}$, that is the diuiſour. Nowe I ſhall diuide $\frac{1}{4}$ by $\frac{3}{16}$, and the quotient will bee $\frac{16}{12}$, that is, 1 $\frac{1}{3}$, wherebу I knowe that there was put 1 ℔ and $\frac{1}{3}$ of ſiluer into the

Crowne, and ſo muche golde taken out for it.

Maiſter. Proue it nowe by examination according to the queſtion.

Scholer. If there were 1 pound $\frac{1}{3}$ of Siluer, then was there of Golde 6 pound $\frac{2}{3}$. Nowe ſay I by the rule of proportion; if eyght pound of Golde expell two pounde of water, what ſhall 6 pound $\frac{2}{3}$ expell?

8 — 2
6 $\frac{2}{3}$ — 1 $\frac{2}{3}$

It will be one pounde $\frac{2}{3}$.

8 — 3 $\frac{1}{2}$
1 $\frac{1}{3}$ — $\frac{7}{12}$

Agayne, if 8 li of Syluer expell 3 li $\frac{1}{2}$ of water, what ſhall 1 $\frac{1}{3}$ expell? It wyll bee $\frac{7}{12}$. Nowe muſte I adde together 1 li $\frac{2}{3}$, and $\frac{7}{12}$ & they will make 2 li, $\frac{3}{12}$, that is 2 li $\frac{1}{4}$, according to the ſuppoſition of the queſtion, wherby I perceyue the worke to be well done. And as I can not but muche reioyce of this excellent inuention, ſo my deſire is kindeled behemently to be perfectly inſtructed in euery part therof, and namely in this poynt, whether the proportion betwene water & gold be ſuch, that for 8 lb of golde put into a veſſell ful of water, there ſhall run out 2 li of water: & foraſmuche ſiluer, whether 3 lb $\frac{1}{2}$ of water would auoide?

Mayſter. I perceyue your meanyng, and coniecture youre imagination to bee thus:

that

that if you knewe the exacte proportion be-tweene Golde and Syluer and Water bothe in their waight and in their quantities, then coulde you easilye finde out the mixtures of them, whiche thinge I haue reserued for an other worke that intreateth suche matters specially. And at this tyme you must consider, that you learne Arithmetike, whiche intreateth of the manner to solue doubtfull questions touching number, without regarde what matter is signified by that number, els were it necessarye in Arithmetike to teache all artes, seeing in it may bee moued questions of all artes. But seing you are so desirous to knowe this thing, I will tell it you in suche a sorte, that you shall practise your arte in finding it, & propounde it in forme of a question. Golde beareth greater proportion to water than Syluer dothe, and their two proportions be in proportion together as $\frac{48}{25}$. But to helpe you somwhat in this riddle, you shall note that ye proportion of quicke Syluer vnto water, is the iuste middle number proportionall in Progression Geometricall, betweene the proportions of Golde and Siluer vnto water. And his proportion is, as $\frac{290}{21}$. Now if you will know the iuste numbers of these 3 proportions, then must

question
the pro-
ortion of
olde, Siluer
d quicke
yluer vnto
vater.

must you finde out 3 numbers in Progression Geometricall, whereof the midlemost must be $\frac{200}{21}$, and the firste muste bee unto the laste, as 25 to 48. And thus I will leaue you to finde those numbers when you be at leysure.

Scholer. Yet sir I thanke you hartily for this much, for nowe I see the possibilitye to finde them out. How be it bicause this questi-on seemeth straunge, if it might please you to instruct me somewhat in the order of working for it, I should the more easily finde the trewe working.

Mayster. You desire to much ease if you will studye for nothing: therefore to occasion you to study the better, I will leaue this doubt wholy to your owne search. But as touching the generaltie of the rule, Archimedes needed not to take two masses of golde and siluer e-quall in waight with the crowne, for the pro-portion might as well bee founde in any other waight, yea although the masse of golde were of one waight, & ye masse of siluer of another. As for example. If the crowne were of 8 pounde waight, as I did suppose, and I haue not so much other fine golde, but onely 1 ℔, and trying that by water, and finding that it doth expell but $\frac{3}{4}$ of an unce of water, yet then by

it may I inferre, that 8 pounde of golde woulde expell 6 vnces of water. And likewayes of the siluer: whereof if I had but 2 pounde, and finde that it doth expell .iij. vnces of water, then might I affirme ꝑ 8 pounde woulde expell 12 vnces, that is 1 ℔ weight. And so is it as good as if the 3 masses were all of one weight. And thus for this tyme I will make an ende of this other parte of Arithmetike.

Scholer. Although I can not sufficiently thanke you for this, yet your promise made me to looke for the arte of Extraction of rootes, whereof hitherto I haue learned nothing.

Maysler. I will not breake my promise, but intend (God willing) to performe it within these three or foure monethes, if I perceyue this my paines to be well taken in the meane season. And you shall not repente the tarying for it, for it shall be encreased by the tarying. And in the meane time, you shall take this Addition not for the second parte of Arithmetike which I promised, but for an augmentation of the first parte, vnto which I woulde haue annexed the extraction of Rootes, square and cubike, namely for Examples of the Statute of Assise of woode, but that in the seconde parte I muste write of diuers other Rootes,

and

and therefore thought it beste to reserue those Rootes also with their examples to the same seconde part.

Scholer. Sir, although I cannot recompence your goodnesse, yet I shall alwaies doe mine indeuour to occasion you not to repente your benefite on me thus imployed.

Mayster. That recompence is sufficient for your part.

FINIS.

The Valuation of Englishe, Flemishe *and French mony, and how eche of them may easily be brought to other value.*

How briefly to reduce. ℔, ß, *and* d. *Flem. into* ℔, ß, *and* d. *English, or Sterling.*

It is to be noted, that 7 pound Flemishe maketh but 6 ℔. sterling, 7 ß flem. maketh 6 ß sterl. and 7 d. fl. 6 d. ster. So that 7 yeldeth but 6. Wherein is euident that there is losse $\frac{1}{7}$, (if it may be so called) when it is reduced into Englishe mony. Wherefore to knowe howe

much 233 li. 13 ß, 4 d, fl. maketh englishe, you muste subtract from it $\frac{1}{7}$, beginning with the poundes, &c. and that which resteth after this subtraction, is the summe required: so that 233 lb, 13 ß, 4 d fl. maketh 200 lb, 5 ß, 8 $\frac{4}{7}$ d. sterling. *Example.* *Another example.*

lb.	ß.	d.		lb.	ß.	d.
233	13	4		311	0	0
$\frac{1}{7}$ 33	7	7 $\frac{3}{7}$	$\frac{1}{7}$	144	8	6 $\frac{6}{7}$
200	5	8 $\frac{4}{7}$ ster.		266	11	5 $\frac{1}{7}$

To reduce lb. ß. and d. ster. into lb. ß. d. flem.

Note that a li, sterling maketh 1 li, 3 ß, 4 d, flem. that is 1 lb $\frac{1}{6}$: 1 ß ster. maketh 1 ß $\frac{1}{6}$ flem. and 1 d. ster. maketh 1 d $\frac{1}{6}$ fl. So that there is gained (if it may so be called) $\frac{1}{6}$ of the summe being thus reduced to fl. For of $\frac{6}{6}$ is made $\frac{7}{6}$, which is 1 whole and $\frac{1}{6}$. Then to know how much 237 lb, 7 ß, 6 d ster. maketh flem. subtract from your ster. the $\frac{1}{6}$ of the whole summe, and adde it to the same summe, and it maketh 276 lb, 18 ß, 4 d ster. which is the summe required. *Example.* *Another example.*

lb.	ß.	d.		lb	ß	d.
237	7	6		337		
39	11	3	$\frac{1}{6}$	56	3	4
276	18	9 sterl.		393	3	4

Ye

Ye shall note, that the equality of Flemishe and Frenche mony is this, that is to say, the li flem. maketh 7 li $\frac{1}{5}$ frenche or tournois: 1 s fle. maketh 7 s $\frac{1}{5}$ frenche, and a grote flem. maketh 7 d $\frac{1}{5}$ frenche.

Wherefore to knowe howe muche 143 li, 4 s 9 d flem. maketh fr. yee must multiplye the whole number twice by 6, beginning at d. and so forwarde: and the produncte of youre seconde multiplication, diuide by 5, so that worke is finished. Or multiply the sayde summe by 7, and take out of it $\frac{1}{5}$ adding it to ye producte of your multiplication by 7, and that is youre number required. So that as well by the one as by the other, 143 li, 4 s, 9 d, flemishe, maketh 1031 li, 6 s, 2 d $\frac{2}{5}$ frenche, or tournois.

Example. *An other example.*

li.	s.	d.		li.	s.	d.
143	4	9 flem.		143	4	9
		6				7
859	8	6		1002	13	3
		6		$\frac{1}{5}$ 28	12	11 $\frac{2}{5}$
$\frac{1}{5}$ 5156	11	0		1031	6	2 $\frac{2}{5}$
1031	6	2 $\frac{2}{5}$ fre.				

An other example.	or thus.
143 lb. flem.	143
6	7
858	1001
6	$\frac{1}{5}$ — 28 12
$\frac{1}{5}$ — 5148	1029 lb. 12 fr.
$\frac{1}{5}$ — 1209 lb $\frac{3}{5}$, or 12 s fren.	

A briefe Reduction of lb s and d French into lb. s. and d Flemish.

Multiplye 233 li, 8 s, 4 d, fr. by 5, and diuide the product twice by 6, that is the sayde number by 6, and the product agayne by 6: and the quotient of this seconde diuision is the thing required. So that 233 li, 8 s, 4 d, fren. maketh 32 lb, 8 s, 4 d, $\frac{5}{9}$ flemishe.

Example.			An other.		
lb.	s.	d.	lb.	s.	d.
$\frac{1}{5}$ — 233	8	4 fren.	753		fren.
		5	5		
$\frac{1}{6}$ — 1167	1	8	3765		
$\frac{1}{6}$ — 194	10	3 $\frac{1}{3}$	$\frac{1}{6}$ — 627.	10	
$\frac{1}{6}$ — 32	8	4 $\frac{10}{18}$, or $\frac{5}{9}$	$\frac{1}{6}$ — 104	11	8

To

To reduce li. ß. and d. Sterling, into li. ß. & d. Frenche or Tournois.

The li ster. maketh 8 li, 8 ß frenche, that is to say, 8 li $\frac{2}{5}$: the ß maketh 8 ß $\frac{2}{5}$, and the peny 8 d $\frac{2}{5}$ frenche. Wherfore to knowe what 231 li, 13 ß, 4 d ster. maketh fren. ye muste multiply your whole summe by 42, that is by 7, and ye product of it by 6, and diuide this second product by 5, and that is the summe required. Otherwise multiply the summe ster. by 8, and adde twise to the product $\frac{2}{5}$, and it shall produce the summe required. So that bothe wayes, 231 li, 13 ß, 4 d ster. maketh 1946 li frenche. As here vnder foloweth.

li.	ß	d.		li.	ß.	d.	
231	13	4	ster.	231	13	4	ster.
		6				8	
1390	0	0		1853	6	8	
		7		46	6	8	
$\frac{1}{5}$ - 9730	0	0		46	6	8	
$\frac{1}{5}$ - 1946	0	0	fren.	1946	0	0	fr.

(Right-hand columns headed: *the same otherwise.*)

An other example.			The same.		
	753	ster.		753	
	6			8	
	4518			6024	
	7		$\frac{1}{5}$	150	12
	31626		$\frac{1}{5}$	150	12
$\frac{1}{5}$	6325 4	fr.		6325 4	fren.

To reduce ℔, ß, and d. fr. into ℔, ß, and d. ster.

To know how much 1256 ℔, 12 ß, 6 d. fr. maketh in sterling money, multiply the summe by 5, and diuide the product by 7 & 6 at twice, and the laste quotient shalbe the thing required, that is to say, 1256 ℔, 12 ß, 6 d., maketh 149 ℔, 11 ß, 11 d. $\frac{4}{7}$ sterling.

	Example.				An other example.		
	℔.	ß.	d.		℔.	ß.	d.
	1256	12	6 ster.		2531		fren.
			5		5		
	6283	2	6		12755		
$\frac{1}{6}$	1047	3	9	$\frac{1}{6}$	2109	3	4
$\frac{1}{7}$	149	11	11 $\frac{4}{7}$	$\frac{1}{7}$	301	6	2 $\frac{2}{7}$

Note that when any mony is giuen by exchange at London for Roan, at 71 d $\frac{1}{2}$: or rather 71 $\frac{3}{7}$, for the crowne of 50 ß frenche. There is

is neyther gayne nor losse, for it is one mony for an other, accompting 8 li, 8 ß fren. for 1 li sterling. So the gyuer loseth the tyme of payment, which is aboute 15 dayes, & he that taketh it, hath the gaine of the same.

They of Roan, that put forthe or take mony by exchange for London, ought to haue like consideration.

Item, when any man giueth at Lõdon 64 d $\frac{1}{7}$, or rather 64 d $\frac{2}{7}$, to haue at one of ye fayres of Lyons a crowne de marc, he that so gyueth his mony, loseth the time, and he that taketh it gayneth the same: for 62 d $\frac{2}{7}$ is equall in value to 45 ß frenche. He that putteth or taketh mony at Lyons for London, ought to consider the same. A crowne de marke, is 45ß.fr. or torn.

Item, when any deliuer in Andwerpe 75 d, to receyue at Lyons à crowne of marcke, hee that putteth it forth loseth the time, and he that taketh it, gayneth the same. For 75 grotes fl. is equall in value to 45 ß frenche.

Thus I make an ende of the practise for exchange, and instruction thereto belonging, minding to shewe the gentle Reader sundry other necessary practises for weight and measures of sundry Countries and townes.

The equalitie, or valuation, weyghtes and measures of sundry Countries and Townes, as foloweth.

ANDWARPE.

An hundreth elles Andwarpe measure, maketh 60 elles of London measure cōmonly vsed in measuring of linnē clothe, & such like, & for the measure of all other things commonly sente, 75 Verges or yardes of Siuille, or other places of Spayne 81 Varres, in Lisbone 60, in the Isle of Madera 62, in Lions 60 elles, in Paris 57, at Roan 52, at Mylan 138 braces, at Genes 288 ½ palmes, at Venis for wollen and linnen cloth 108 braces, at Florens 122 ½ braces, at Luke 120, at Nuremberg 104 elles, at Franckforde, Leypzig and Preslawe 125 elles, at Dansicke 83, at Vienna in Austria 87.

It is to be noted, that all silkes be bought at Andwerpe by the Bruges elle, whiche is greater than the common measure, (by which they retayle) by 2 in the C.

Item 100 lb. at Andwerpe, maketh at London 104, at Francforde 92 lb, at Collen and Augsburg 95, at Nuremberg 93, at Roan 91, at Lyons betwixte 110 and 111, at Paris 95, at Diepe 93, some holde 90, at Amsterdā 95, at Geneua 84, at Toulouse 114, at Rochell 116, at

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at Merseille 115, at Seuille and other places of Spaine 102, at Venice, a la Sotilla, or small wayghth 155, by grosse wayghte 98, at Aquila in Abruzzo 146, at Vienna 83, at Preslau 125, at Leypzig 94, at Dansicke 120, at Lubec, 98½, at Barcellone 133⅓, at Lisbone smalle wayght 92, great wayght 86, at Gennes 146.

At Andwerpe, Golde and Siluer is weyghed by the Mark, the marke is 8 vnces, the vnce 20 estrelins, and the estrelin 32, as our graynes, ye goldesmithes diuise that into smaller weyght, but not the Marchantes. The proofe of Golde is made by Karactes, whereof 24 maketh a marke of fine golde, the karacte is 24 graynes, the proofe of the money is made by denieres, whereof 12 d. is 1 ß fine, that is a marke of fine siluer, the d. is also diuided into 24 graynes, and the grayne into 4 quarters.

Item 100 markes in Andwerpe Troy weyght, maketh at Lyons 103 markes, 2 vnces, and 20 graynes 23 p̈. at Nuremberge 103 markes 2½ vnces, 2 quints, 3 d., at Francfort 105 marks, at Augsburg 104 marks, 3 vnc. 1 quint, at Venice 103 markes, 1 vnce, 7 d., 18 graynes, at Bourges in Spayne 116⅔, at London 66 lb.

The Marke of siluer or golde at Andwerpe Troy weight, which is 8 vnces, maketh 7½ vnces common weight, with which all other marchandise

marchandise is weyghed, so that the Troy weyght is greater than the common by 6 ½ in the C. By this weyght of Troy, they weygh Muske, Amber and Pearle. &c.

LONDON.

Of all kinde of marchandise solde by weight at London, they sell or buy alwayes 112 lb. for the hundred, for half an hund. 56 lb. and so after the rate; but in bying or selling by the lb, they giue but 1 lb. for all, to witte 16 ounces.

Golde and Siluer is solde by the lb, contayning 12 vnces, which weyghe more than ye common vnces: for 100 markes, at 8 vnces to the marke, maketh at Lions or Andwerpe 104, at Nuremberge 103 markes.

A hundred lb common weyght at London, maketh at Andwerpe 96: at Lyons 106: at Roan 90: at Millan 142. And of silke weyght 154, at Vincence silke weight 130, at Marseille 116 ½, at Francforte and Nuremberge 89, at Rochell and Marseille 112 or 114, at Paris 91, at Venise a la Sotilla 151, at Castill and other places in Spayne 102: moreouer 96 lb Saffron weyghte, maketh at Francfort and Nuremberge 100 lb. At London is vsed two sortes of measures, to wytte the elle and the yarde, by the

Bond

[illegible] [illegible] Vaux [illegible]